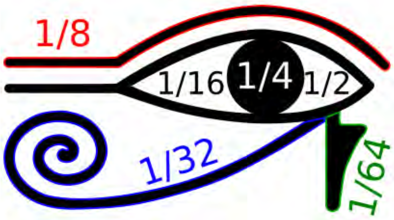
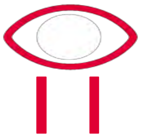
Fractions

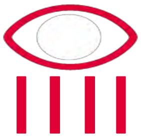
Did you know?



5000 years ago the Egyptians used fractions but could only write unit fractions, fractions where the numerator is 1.

$\frac{1}{2} =$ 

$\frac{1}{3} =$ 

$\frac{1}{4} =$ 

All other fractions were written as sums of unit fractions but more about that later………

Fractions 1

1. What is the value of $\frac{2006}{8} + \frac{6002}{8}$

2. There are 84 animals in a field
   11 are cows
   45 are sheep
   The rest are pigs

   What fraction of the animals are pigs? Give you answer in simplest form

3. Simplify fully $\frac{x}{6} + \frac{3x}{4}$

4. Calculate $\frac{5}{6} \times \frac{3}{5}$

   give your answer in simplest form

5. What is the value of

6. How many of these calculations equal 1
   Give reasons

   $\frac{1}{2} + \frac{1}{2}$ $\quad$ $\frac{1}{2} - \frac{1}{2}$ $\quad$ $\frac{1}{2} \times \frac{1}{2}$ $\quad$ $\frac{1}{2} \div \frac{1}{2}$

7. Sally has 30m of ribbon.
   She cuts lengths of $2\frac{3}{5}$ metres from the ribbon. Sally says she has enough ribbon to cut 12 lengths. Is she correct? You must show all workings

8. Express as a single fraction $\frac{2a}{3} - \frac{b}{4}$

You can do the next section for fun or move on if you got all of Fractions 1 correct

Fractions 2

1. Calculate $2\frac{1}{7} + 1\frac{1}{5}$
   Give your answer as a mixed number in simplest form

2. Simplify $\frac{4a}{5}$ x $\frac{7b}{3}$

3. Work out $\frac{19}{24} - \frac{3}{8}$
   giving your answer in simplest form

4. Find the mean of $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ and $\frac{1}{6}$
   give your answer in simplest form

5. A full glass of water can hold $\frac{1}{6}$ of a bottle of water.

   How many glasses can be filled by $2\frac{1}{5}$ bottles?

6. A water tank is $\frac{2}{3}$ full
   40 litres of water are taken from the tank
   The tank is now $\frac{1}{2}$ full
   What fraction of the tank was removed?

7. Which of these has the largest value

   $\frac{1}{2} + \frac{1}{4}$ $\quad$ $\frac{1}{2} - \frac{1}{4}$ $\quad$ $\frac{1}{2} \times \frac{1}{4}$ $\quad$ $\frac{1}{2} \div \frac{1}{4}$ $\quad$ $\frac{1}{4} \div \frac{1}{2}$

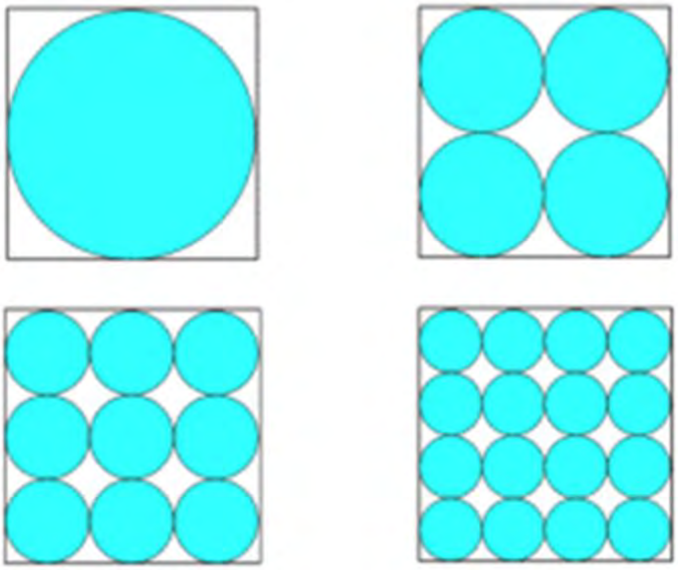
8. Simplify $\frac{a}{b} + \frac{b}{c}$

Four Short Problems

Circles

In the images below the square has a side length of 1 unit

Which of the images has the greatest area covered by the circles?



Petrol Tank

Andrea’s car has a petrol tank that holds 44 litres of petrol.

She goes to the petrol station when her tank is a quarter full and fills it up until it is two thirds full.

How many litres of petrol does she put into the car’s petrol tank?

Peaches

A monkey has 75 peaches

Each day he: keeps a fraction of his peaches
gives some away
eats 1 peach

These are the fractions he decided to keep.

$$\frac{1}{2} \quad \frac{1}{4} \quad \frac{3}{4} \quad \frac{3}{5} \quad \frac{5}{6} \quad \frac{11}{15}$$

In what order did he use the fractions so that he was left with just one peach at the end?

Integers

What is the integer x

so that $\frac{x}{9}$

lies between $\frac{71}{7}$ and $\frac{113}{11}$?

Fractions of 1000

What is $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{7}{8}$ of $\frac{8}{9}$ of $\frac{9}{10}$ of 1000 ?

Unit Fractions

Remember the Egyptians and their unit fractions? Now it's time to explore this further……

A unit fraction is a fraction that has a numerator of 1.

Other fractions can be written as the sum of two unit fractions.

Here are some examples some of which are correct and some which are not – can you find which ones are correct?

$$\frac{1}{2} = \frac{1}{3} + \frac{1}{6} \qquad \frac{1}{2} = \frac{1}{10} + \frac{1}{20} \qquad \frac{1}{3} = \frac{1}{4} + \frac{1}{12} \qquad \frac{1}{3} = \frac{1}{7} + \frac{1}{21} \qquad \frac{1}{4} = \frac{1}{5} + \frac{1}{20}$$

What rules might have been used to generate these?

Thinking about those that are correct what rule might you suggest for generating other unit fractions from the sum of two others?

Some unit fractions can be made in more than one way …..

Here are some to start you off $\frac{1}{6} = \frac{1}{7} + \frac{1}{42}$ $\frac{1}{6} = \frac{1}{8} + \frac{1}{24}$ can you find more ways to make $\frac{1}{6}$?

Can you finish this sum for $\frac{1}{8}$ and find more? $\frac{1}{8} = \frac{1}{9} + \frac{1}{}$

Can all unit fractions be made in this way? Choose different unit fractions to test out your theories.

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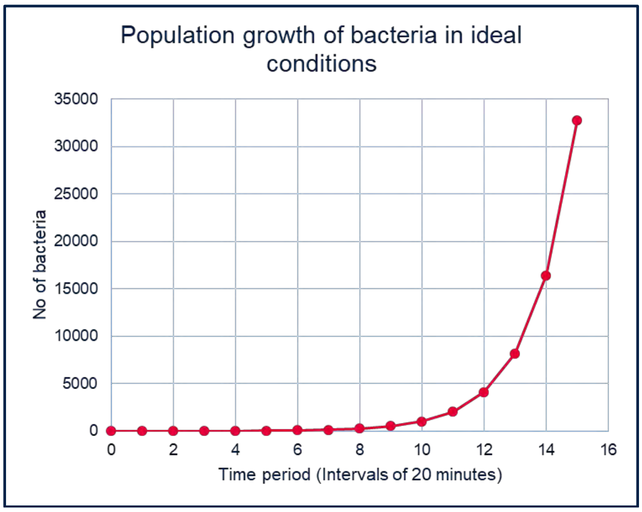
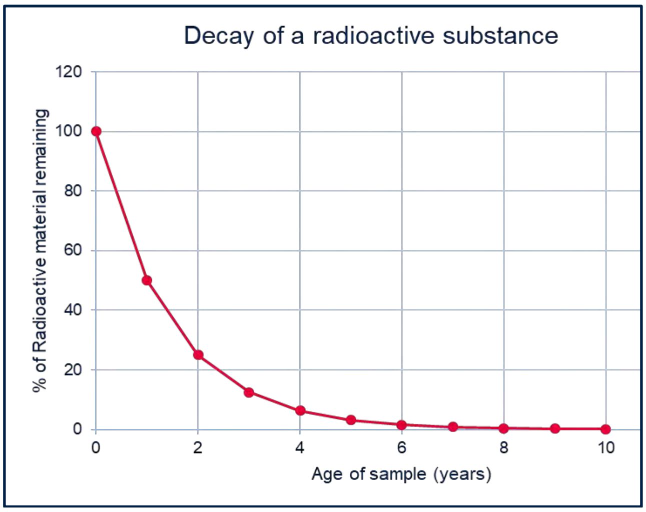
Indices

Did you know?

Indices are also referred to as exponents





e.g. $2^{\textcircled{3}} = 8$

3 is the 'exponent'

$$2^3 = \overbrace{2 \times 2 \times 2}$$

It tells us how many times a number is multiplied by itself

This is where exponential graphs come from!

Indices 1

Simplify the following

1. $x^3 \times x^8 =$

2. $\frac{9^8}{9} =$

3. $(2^3)^5 =$

4. $\frac{4^4 \times 4}{(4^2)^3} =$

5. $16^{\frac{1}{2}} =$

6. What is the reciprocal of 16

7. What is 4^{-3}

8. What is $\left(\frac{2}{5}\right)^{-1}$

Indices 2

Simplify the following

1. $t^5 \times t^4 =$

2. $\frac{8^7}{8^2} =$

3. $(3^4)^2 =$

4. $\frac{5^7 \times 5}{(5^3)^3} =$

5. $(8)^{\frac{1}{3}} =$

6. $y^0 =$

7. What is $4^{-3} =$

8. What is $\left(\frac{2}{3}\right)^{-2} =$

Roots and Indices Maze

Can you find the way from one side of the table to the other?

- Begin in the highlighted box
- Move vertically or horizontally one box at a time ……..no diagonal moves allowed
- You may only land on boxes which are equivalent in value to the highlighted one

2^6x2^3	3^2x2^3	$(\sqrt{16})^2$	$(2^3)^3$	$8^3\div8$	4^4x4^{-3}	$(\sqrt[3]{8})^4$	$8x4^2$
$\sqrt{8}^3$	$(2^3)^2$	8^7x8^{-5}	4^3	$2^{-2}x2^7$	64^0	2^5x2^3	$4^7\div2^3$
$(\sqrt{64})^3$	8^2	2^2x2^3	2^3x2^3	$(2^3)^3$	$(\sqrt[3]{8})^6$	4^6x4^{-3}	2^2x4^2
2^6	$(\sqrt{64})^2$	4^6x4^{-2}	$(\sqrt{16})^3$	$(2^2)^4$	$8^3\div2^3$	$2^{-3}x2^7$	$(2^2)^4$
3^5	2^6x2^1	8^3	$4^5\div2^4$	$(-4)^{-3}$	$(2^2)^3$	$(\sqrt{8})^3$	$4^6\div2^6$
4^3x4^{-3}	$(2^5)^1$	$(\sqrt[3]{64})^2$	2^3x8	$2^{-1}x2^7$	$(\frac{1}{4})^{-3}$	16^2	64

Hint : What is the value of 2^6

Matching Pairs

Match the expressions in Column A with their equivalent expression in Column B

A	B
$\left(\frac{9}{16}\right)^{\frac{1}{2}}$	$\frac{3}{2}$
$(4)^{\frac{3}{2}}$	8
$(-5)^{-2}$	$\frac{1}{16}$
$(16)^{-\frac{3}{2}}$	$\frac{1}{4}$
$(2)^{-3}$	$\frac{3}{4}$
$(64)^{-\frac{1}{3}}$	$\frac{1}{25}$
$\left(\frac{4}{9}\right)^{-\frac{1}{2}}$	$\frac{1}{8}$
4^{-2}	$\frac{1}{64}$

Where does it belong?

Five numbers are arranged below in order from least to greatest

$$x, \quad x^3, \quad x^4, \quad x^2, \quad x^0$$

- Where does $-x^{-1}$ belong in the list above?

Hints

- The numbers are arranged in order ($x < x^3 < x^4 < x^2 < x^0$)
- When is a cubed number greater than a squared number?
- Are there any of the terms that you know the value of ?
- Draw a number line and try some values in the expressions – what happens?

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Surds

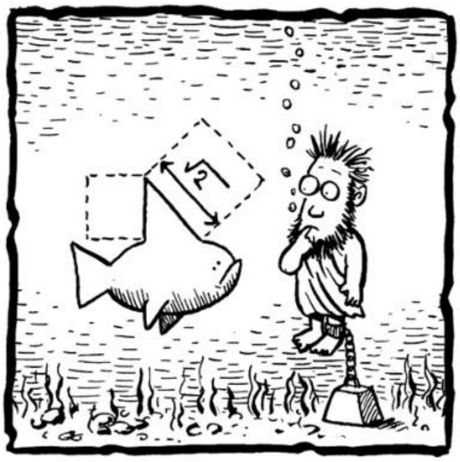
Did you know?

Maths can be murderous!

You will have heard of Pythagoras and his theorem but have you heard of Hippasus who was one of his followers?

Pythagoreans preached that all numbers could be expressed as the ratio of integers – i.e. fractions.

Hippasus is sometimes credited with the discovery of the existence of irrational numbers – proving it for $\sqrt{2}$. Following which, he was drowned at sea!



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Surds 1

1. Simplify $\sqrt{a} + 2\sqrt{a} + 5\sqrt{a}$

2. Simplify $\sqrt{2} \times \sqrt{6}$

3. Simplify fully $(4\sqrt{3})^2$

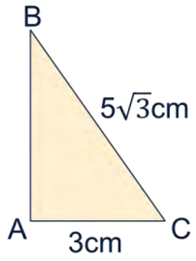
4. Write $\sqrt{45} + \sqrt{20}$ in the form $k\sqrt{5}$

5. Calculate $\frac{\sqrt{54}}{\sqrt{6}}$

6. Rationalise the denominator of $\frac{4}{\sqrt{3}}$

7. A rectangle has an area of $8\sqrt{15}$ cm^2 and a length of $2\sqrt{3}$ cm.
   Find the width of the rectangle

8. Find the length AB



Surds 2

1. Simplify $\sqrt{d} + 6\sqrt{d} - 3\sqrt{d}$

2. Simplify $2\sqrt{b} \times 4\sqrt{3}$

3. Simplify fully $(4\sqrt{5})^2$

4. Write $\sqrt{75} + \sqrt{48} - 2\sqrt{12}$ in the form $k\sqrt{3}$

5. Simplify $\frac{\sqrt{125} - 2\sqrt{20}}{\sqrt{5}}$

6. Rationalise the denominator of $\frac{2\sqrt{2}}{\sqrt{5}}$

7. Evaluate $\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{\sqrt{6}}$

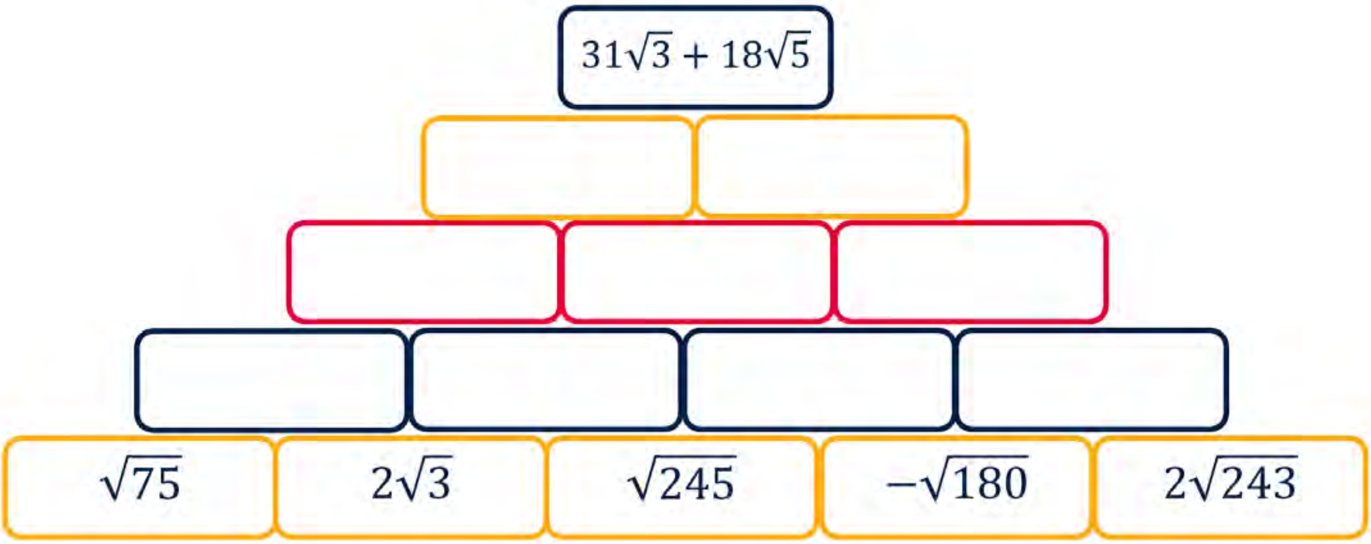
8. A triangle has base of $3\sqrt{2}$ and a perpendicular height of $5\sqrt{8}$

   Calculate the area of the triangle

Another Brick in the Wall

Complete the empty boxes in the pyramid.

Each box is the sum of the two boxes directly below it.



Hint: You may need to simplify some of the surds in the bottom row to get started.

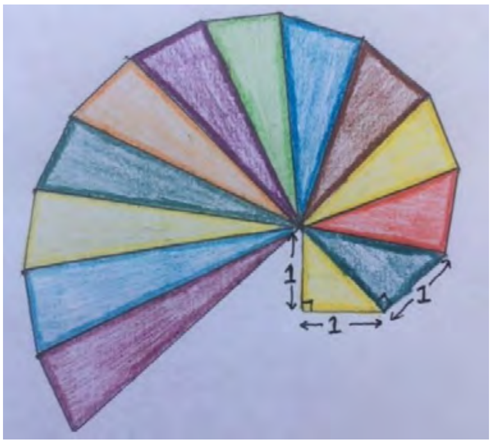
True or False

Decide if each of the following expressions is *True* or *False*

1. $\sqrt{9} + \sqrt{4} = \sqrt{13}$
2. $\sqrt{a} \times \sqrt{b} = \sqrt{c}$
3. $\sqrt{(8)^2} = 8$
4. $10\sqrt{2} = \sqrt{8}$
5. $\frac{\sqrt{12} \times \sqrt{3}}{\sqrt{9}} = 2$
6. $\sqrt{2}^3 = 2\sqrt{2}$
7. $\sqrt{ab}^2 = ab$
8. $2\sqrt{100} = \sqrt{200}$

Are there some statements that are ‘Sometimes true’ but not ‘Always true’? Explain why.

The Wheel of Theodorus



The diagram shows a spiral made up of right angled triangles.

The shortest side of each triangle measures 1 unit long.

Can you see how it is constructed?

Find the length of the hypotenuse of the first few triangles.

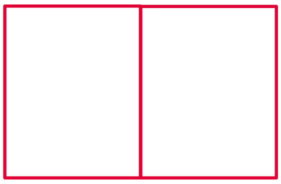
What do you notice ?

Which triangle would have a side length of 3?

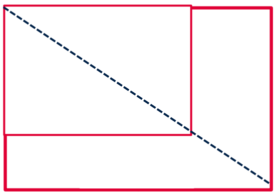
What other questions might you want to ask about the diagram?

Take a sheet of A4 paper

Take a sheet of A4 paper – actually take 2 and place them side by side to make A3 paper. Like this

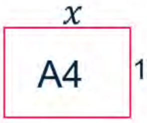


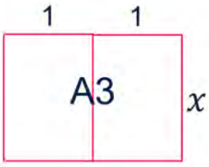
Then place another A4 piece of paper on top of it like this



What do you notice about the ratio of the sides of an A3 sheet to an A4 sheet?

Thinking about the ratio of the long side to the short side we get





$$\frac{x}{1} = \frac{2}{x} \rightarrow x^2 = 2 \rightarrow x = \sqrt{2}$$

Therefore, for A4, A3, A2, etc… the length of the long side divided by the length of the short side is always $\sqrt{2}$

Simplifying solutions

Fractions

Fractions 1

1. 1001
2. $\frac{1}{3}$
3. $\frac{11x}{12}$
4. $\frac{1}{2}$
5. $\frac{16}{7}$
6. $\frac{1}{2}+\frac{1}{2}\ and\ \frac{1}{2}\div\frac{1}{2}$
7. $No, 31\frac{1}{5} > 30$
8. $\frac{8a-3b}{12}$

Fractions 2

1. $3\frac{12}{35}$
2. $\frac{28ab}{15}$
3. $\frac{5}{12}$
4. $\frac{5}{16}$
5. $13\frac{1}{5}, will\ fill\ 13\ glasses$
6. $\frac{1}{6}$
7. $\frac{1}{2}\div\frac{1}{4}=2$
8. $\frac{ac+b^2}{bc}$

Circles *All the shaded areas are equal to* $\frac{1}{4}\pi$

Peaches *On Day 6 the monkey keeps* $\frac{1}{4}$ *of 8 (2 peaches) gives 6 away and eats 1 leaving 1 at the end. Order:* $\frac{11}{15}, \frac{5}{6}, \frac{3}{4}, \frac{1}{2}, \frac{3}{5}, \frac{1}{4}$

Petrol Station Solution $18\frac{1}{3}$ *litres*

Integers solution $x = 92$

Fractions of 1000 Solution $\frac{1}{10}$ x 1000 = 100

Unit Fractions Solutions

$\frac{1}{2}=\frac{1}{3}+\frac{1}{6}$ $\quad$ $\frac{1}{2}=\frac{1}{10}+\frac{1}{20}$ $\quad$ $\frac{1}{3}=\frac{1}{4}+\frac{1}{12}$ $\quad$ $\frac{1}{3}=\frac{1}{7}+\frac{1}{21}$ $\quad$ $\frac{1}{4}=\frac{1}{5}+\frac{1}{20}$

In general this can be written as: $\frac{1}{n}=\frac{1}{n+1}+\frac{1}{n(n+1)}$

Some unit fractions can be made in more than one way

Simplifying solutions

Here are some examples for $\frac{1}{6}$ and $\frac{1}{8}$

$\frac{1}{6}=\frac{1}{9}+\frac{1}{18}$ $\frac{1}{6}=\frac{1}{10}+\frac{1}{15}$ $\frac{1}{6}=\frac{1}{12}+\frac{1}{12}$

$\frac{1}{8}=\frac{1}{9}+\frac{1}{72}$ $\frac{1}{8}=\frac{1}{10}+\frac{1}{40}$ $\frac{1}{8}=\frac{1}{12}+\frac{1}{24}$

Unit fractions with denominators which are prime can only be written one way as the sum of two distinct unit fractions so, you can do $\frac{1}{7}=\frac{1}{8}+\frac{1}{56}$ but you cannot find another one for $\frac{1}{7}$

Indices

Indices 1

1. x^{11}
2. 9^7
3. 2^{15}
4. $\frac{1}{4}$
5. 4
6. $\frac{1}{16}$
7. $\frac{1}{64}$
8. $\frac{5}{2}$

Indices 2

1. t^9
2. 8^5
3. 3^8
4. $\frac{1}{5}$
5. 2
6. 1
7. $\frac{1}{81}$
8. $\frac{9}{4}$

Maze

$2^6 \times 2^3$	$3^2 \times 2^3$	$(\sqrt{16})^2$	$(2^3)^3$	$8^3 \div 8$	$4^4 \times 4^{-3}$	$(\sqrt[3]{8})^4$	8×4^2
$\sqrt{8}^3$	$(2^3)^2$	$8^7 \times 8^{-5}$	4^3	$2^{-2} \times 2^7$	64^0	$2^5 \times 2^3$	$4^7 \div 2^3$
$(\sqrt{64})^3$	8^2	$2^2 \times 2^3$	$2^3 \times 2^3$	$(2^3)^3$	$(\sqrt[3]{8})^6$	$4^6 \times 4^{-3}$	$2^2 \times 4^2$
2^6	$(\sqrt{64})^2$	$4^6 \times 4^{-2}$	$(\sqrt{16})^3$	$(2^2)^4$	$8^3 \div 2^3$	$2^{-3} \times 2^7$	$(2^2)^4$
3^5	$2^6 \times 2^1$	8^3	$4^5 \div 2^4$	$(-4)^{-3}$	$(2^2)^3$	$(\sqrt{8})^3$	$4^6 \div 2^6$
$4^3 \times 4^{-3}$	$(2^5)^1$	$(\sqrt[3]{64})^2$	$2^3 \times 8$	$2^{-1} \times 2^7$	$(\frac{1}{4})^{-3}$	16^2	64

Simplifying solutions

Matching Pairs

A	B
$\left(\frac{9}{16}\right)^{\frac{1}{2}}$	$\frac{3}{4}$
$(4)^{\frac{3}{2}}$	8
$(-5)^{-2}$	$\frac{1}{25}$
$(16)^{-\frac{3}{2}}$	$\frac{1}{64}$
$(2)^{-3}$	$\frac{1}{8}$
$(64)^{-\frac{1}{3}}$	$\frac{1}{4}$
$\left(\frac{4}{9}\right)^{-\frac{1}{2}}$	$\frac{3}{2}$
4^{-2}	$\frac{1}{16}$

Where does it belong the order should be as follows $x, x^3, x^4, x^2, x^0, -\boldsymbol{x}^{-1}$

Surds

Surds 1

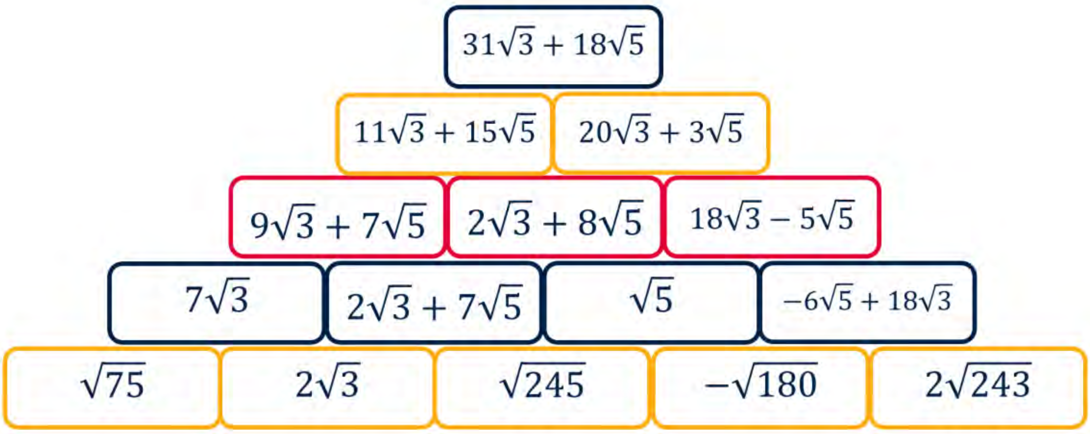
1. $8\sqrt{a}$
2. $2\sqrt{3}$
3. 48
4. $5\sqrt{3}$
5. 3
6. $\frac{4\sqrt{3}}{3}$
7. $\sqrt{66}\ cm$
8. $4\sqrt{5}cm$

Simplifying solutions

Surds 2

1. $4\sqrt{d}$
2. $8\sqrt{3b}$
3. 80
4. $5\sqrt{3}$
5. 1
6. $\frac{2\sqrt{10}}{5}$
7. $\sqrt{2}$
8. $30\ cm^2$

Another Brick in the Wall



$31\sqrt{3}+18\sqrt{5}$

$11\sqrt{3}+15\sqrt{5}$ | $20\sqrt{3}+3\sqrt{5}$

$9\sqrt{3}+7\sqrt{5}$ | $2\sqrt{3}+8\sqrt{5}$ | $18\sqrt{3}-5\sqrt{5}$

$7\sqrt{3}$ | $2\sqrt{3}+7\sqrt{5}$ | $\sqrt{5}$ | $-6\sqrt{5}+18\sqrt{3}$

$\sqrt{75}$ | $2\sqrt{3}$ | $\sqrt{245}$ | $-\sqrt{180}$ | $2\sqrt{243}$

True or False

1. $False$	2. $True\ when\ a \times b = c$ $otherwise\ False$	3. $True$	4. $True$
5. $True$	6. $True$	7. $True$	8. $False$

The Wheel of Theodorus

Can you see how the diagram is constructed?

The hypotenuse of the first triangle then becomes a side on the next triangle.

Find the length of the hypotenuse for the first few triangles.

Using Pythagoras' theorem $a^2 + b^2 = c^2$ we get $1^2 + 1^2 = 2$ so $c^2 = 2$.
Then the hypotenuse of the smallest triangle is $\sqrt{2}$
This means the second triangle has sides with length 1 and $\sqrt{2}$
Using Pythagoras' theorem again gives us the hypotenuse of $\sqrt{3}$ Continuing, the next hypotenuse would be $\sqrt{4}$ (which equals 2), then $\sqrt{5}$ and so on.

What do you notice?

The lengths of the first few hypotenuses are $2, \sqrt{3}, \sqrt{4}, \sqrt{5}$ ...
You will have noticed a pattern in these lengths - so do we really need to keep using Pythagoras' theorem to find the hypotenuse of each of the triangles?

Which of the triangles would have a side length of 3

We know that $\sqrt{9} = 3$ and from the patterns described above we can see there would be a side of length 3 in the 8th (purple) and 9th (orange) triangles.

Expanding

? ?

Did you know?

This illustration shows a visual times-table or multiplication square.

- What do you notice?
- Can you see square numbers?
- Can you see prime numbers?

As you progress through this topic try to think about how this diagram might relate to what you are doing..

Expanding 1

1. Which of the following are the same as:

3(4 + 10)

3 x 4 + 10

(3 x 4) +(3 x10)

3 x 14

3 + 14

2. Which of these are the same calculation?

24 x 14

12 x 2 x 14

6 x 4 x 2 x 14

3 x 7 x 8 x 2

2 x 7 x 6 x 4

3. Expand $4(x+5)$

4. Calculate $\frac{5}{6}\left(2+\frac{3}{5}\right)$

5. Which expression is the odd one out?

(20 x 8) + (4 x 8)

2 x 4 x 2 x 6

(12 x 8) + (12 x 8)

8(20 + 4)

24 x 8

6. Which of these expressions are the same?

$12(x+1)$

$4(3x+2)$

$2(6x+3)$

$6(2x+1)$

7. Expand -3(2y + x)

8. 4 people have (x+3) apples each and 5 people have (x-4) apples each.

Write an expression, in its simplest form for the total number of apples.

A multiplicative string

Have a go at doing the following sum in your head:

3 x 7 x 2 x 4 x 5

Did you multiply from left to right?

Here is an alternative suggestion:

3 x 7 x 2 x 4 x 5
(2 x 5) x ((3 x 7) x 4)
(10) x (21 x 4)
10 x 84
840

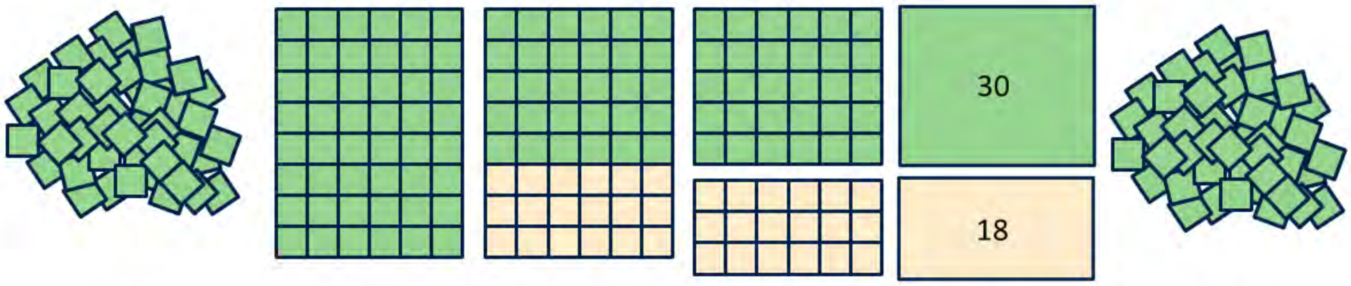
- Why has it been done this way?
- What do the brackets represent?

Multiplication matching

Which of the following multiplicative strings are the same…can you match them into sets?

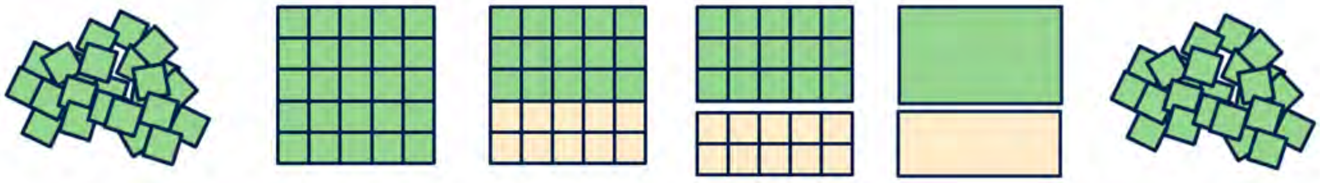
- 9 x 8 x 6
- 2 x 3 x 2 x 4 x 5
- 20 x 4 x 7
- 15 x 4 x 6 x 5 x 7
- 4 x 6 x 5 x 7 x 3
- 3 x 4 x 6 x 3 x 2
- 10 x 21 x 6
- 2 x 5 x 7 x 3 x 6
- 12 x 4 x 5
- 10 x 6 x 4
- 2 x 2 x 5 x 7 x 4
- 35 x 3 x 4 x 6
- 6 x 35 x 6
- 12 x 6 x 6
- 10 x 7 x 8

Geometric Interpretations



48 = 6 x 8 = 6 (3 + 5) =(6 x 3) + (6 x 5) = 18 + 30 = 48

- Use the example above to complete the one below

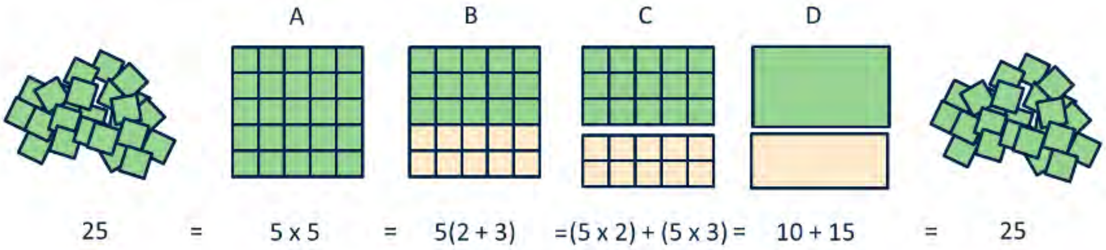


25 = = = = = 25

- Complete the table below in a similar manner:

Total	A	B	C	D	Total
81	9 x 9	9(3 + 6)			
	5 x 12		(5 x 4) + (5 x 8)		
	… x 8			27 + 45	72
	15 x …			… + 30	90
144			(… x 7) + (… x 5)		
		3(… + 4)			36

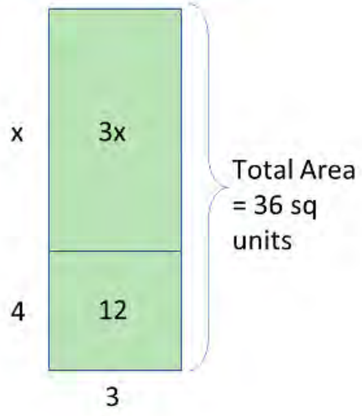
Algebraic Applications



How does what you have been doing relate to the equations below?

Try drawing diagrams to illustrate your explanations

1. $3(x + 4) = 36$ ⟶ e.g.
2. $5(x + 3) = 35$
3. $x(x + 7) = 44$
4. $5(x + y + 4) = 45$



Match it Up!

Square x, then mulipy by three

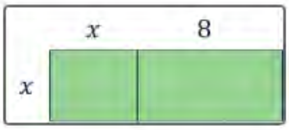
Add twelve to x, then divide by three

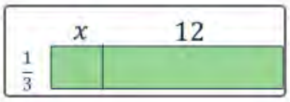
Add eight to x, then multiply by x

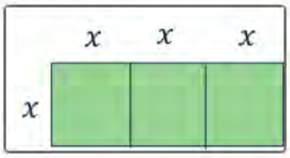
Half x, then add three

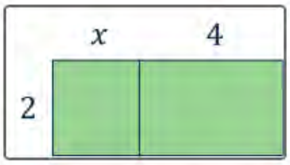
Add five to x, then multiply by x

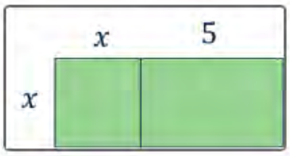
Add four to x, then multiply by two

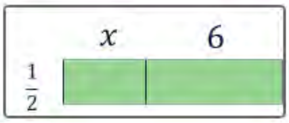












$\frac{x+6}{2}$

$2x+8$

$x(x+5)$

$3x^2$

$x(x+8)$

$\frac{1}{3}(x+12)$

Odd One Out

In each group of four expressions, one of them is not the same as the rest. Can you find it?

- $(3x+4y)+2(x+2y)$
- $4(2x+5y)-3(x+4y)$
- $3(2x+3y)-(x-y)$
- $3(x+3y)+(2x-y)$

- $(x+3)(x+7)$
- $x(x+3)+7(x+3)$
- $x(x+2)+7(x+2)+x+7$
- $x(x+4)+6(x+3)$

- $x(x+3)+3(x+5)$
- $2(x+4)+x(x+4)$
- $(x+3)^2+6$
- $x(x+3)+4(x+3)+(x+3)$

- $x(x-6)-(-2x)-2(x-6)$
- $x(x-6)+2(x-6)$
- $x(x-2)-2(x-2)-2(x-4)$
- $(x-3)^2+3$

Expanding 2

1. Expand y(2y – 3)

2. Expand 2x^2(3xy – 2x^3)

3. Expand and simplify

   a. $5(x-4)+3(2x+5)$

4. Expand and simplify

   a. $4\left(\sqrt{2}-3\right)+2(\sqrt{2}+2)$

5. Multiply the expressions y and y + 4

   Which of these expressions show the result?

   $\boxed{5y}$ $\boxed{y(y+4)}$ $\boxed{y^2+4y}$ $\boxed{4y+4}$

6. A rectangle of width 3cm and width x + 4 cm is made larger by doubling its side lengths. What is the area, in cm^2 of the larger rectangle?

7. Expand and simplify 4 – 3(2 – a + t) - t

8. Expand and simplify

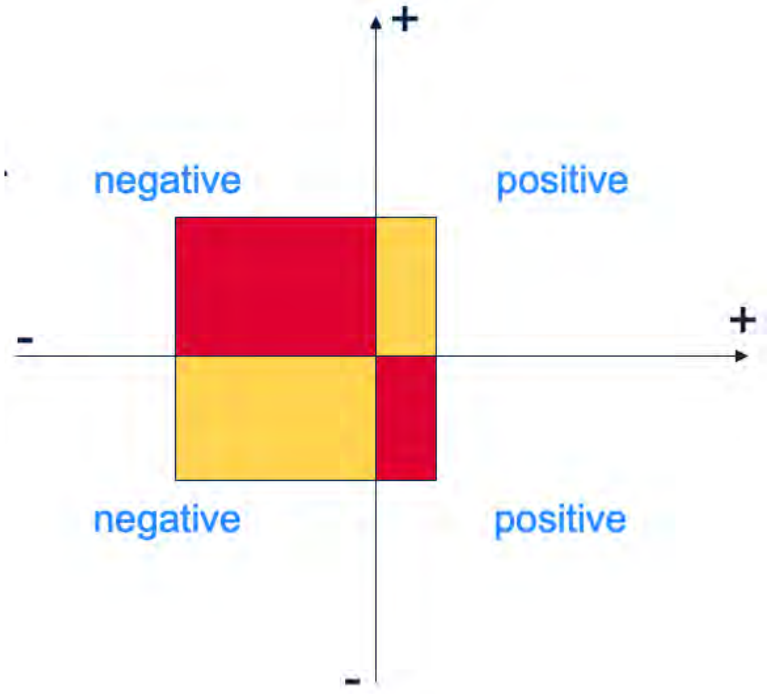
   $\frac{a}{2}\left(3+\frac{a}{4}\right)+\frac{a}{3}(2+\frac{a}{2})$

Double Brackets

Did you know?

We use a 2D co-ordinate plane to develop understanding regarding multiplication and also a *lot* of algebra.

How can the diagram help us understand what happens when we multiply negative numbers?



Dealing with Negative Numbers

1. 4 x (-7) x 6
2. 3 x 9 x (-6)
3. 2 x (-3) x (-4)
4. 2 x (-2) x (-2) x (-5)
5. a x 7 x a
6. ab x 3 x 6b
7. (-4a) x 7a x (-6a)
8. Use what you have noticed to fill in the gaps in the sentences below:

positive | negative | ODD | EVEN

With an …….. number of negative numbers then value will be ……….

With an …….. number of negative numbers then value will be ……….

Expanding 1

1. Without doing the calculation, will the answer to this calculation be positive or negative? Give a reason.
   a. 2 × (-3) × (-4) × 6 × (-6) × (-1) × 7 × (-2)
2. 24 × 17 is the same as which of the following

$2 \times 3 \times 17 \times 2 \times 2$

$(20 + 4)(10 + 7)$

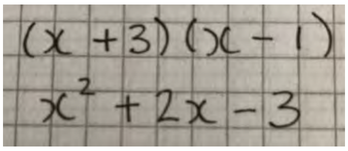
$(30 - 5)(20 - 2)$

$20(10 + 7) + 4(10 + 7)$

3. Expand $3(\sqrt{3} - 6)$
4. Expand and simplify
   1. $(x + 2)(x + 5)$
5. Expand and simplify
   a. $(x + 6)(x - 2)$
6. Expand and simplify
   a. $(\sqrt{2} + 3)(\sqrt{2} + 1)$
7. Expand and simplify
   a. $(x^2 + 2)(x^2 + 6)$
8. Expand and simplify
   a. $(x^2 + 3)(x^3 + 7)$

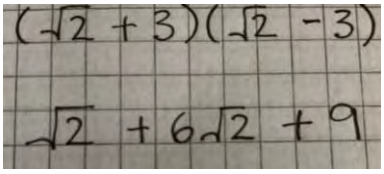
What's gone wrong?

- Here is a student's work on expanding brackets.
- Take a look and decide if they have done the work correctly or not.
- If they have made a mistake can you say why ?
- What are the correct answers?



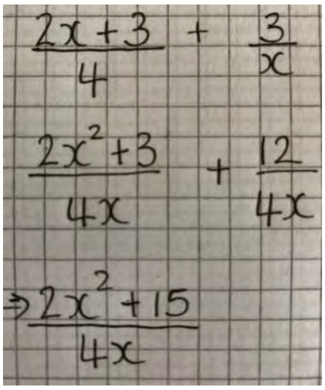
$(x+3)(x-1)$

x^2+2x-3



$(\sqrt{2}+3)(\sqrt{2}-3)$

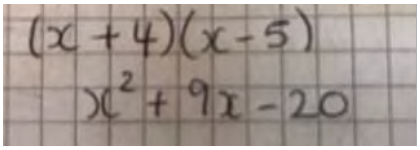
$\sqrt{2}+6\sqrt{2}+9$



$\frac{2x+3}{4}+\frac{3}{x}$

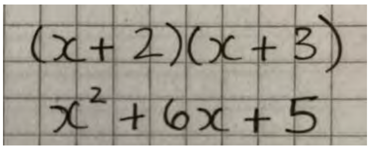
$\frac{2x^2+3}{4x}+\frac{12}{4x}$

$\Rightarrow\frac{2x^2+15}{4x}$



$(x+4)(x-5)$

$x^2+9x-20$



$(x+2)(x+3)$

x^2+6x+5

Expand and Simplify

- Expand the expressions below, then find the matching expression in the grid.
- When completed there should be four answers unmatched.
- Find the sum of these four expressions and simplify it

$x^2+6x-16$	x^2+6x+9	x^2+6x+8	x^2+9
$x^2+7x+12$	x^2-9x+8	$x^2-5x+12$	$x^2-8x+14$
$9-x^2$	$-x^2+6x+36$	$x^2+10x+28$	x^2+x-12

1. $(x+3)^2$
2. $(x+4)(x+3)$
3. $(x-4)^2-2$
5. $(x+5)^2+3$
6. $x(x+4)+2(x+4)$
7. $(3-x)(3+x)$
8. $x(x-8)-(x-8)$

Quadratic Puzzles

- These are multiplication grids
- We can use these to expand quadratics such as $(x+3)(x+4)$

$(x+3)$

$(x+4)$

×	x	$+3$
x	x^2	$3x$
$+4$	$4x$	12

$x^2 + 3x + 4x + 12$

There are 4 terms after expanding the brackets. That's why these expressions are called quadratics – as 'quad' means four.

Now we can simplify by collecting like terms to get this → $x^2 + 7x + 12$

Quadratic Puzzles (cont…)

- Fill in the blanks in the multiplication grids
- Do you notice anything?

$(x-4)$

$(x-4)$

×	x	-4
x		
-4	$-4x$	$+16$

$x^2 \boxed{} + 16$

$(2x+1)$

$(x+2)$

×	$2x$	
x		
$+2$		$+2$

$2x^2$

$(3x-5)$

$(x+3)$

×	$3x$	
x		
	$+9x$	

$(2x+3)$

$(2x-4)$

×		
$2x$		

$(3x+4)$

$(2x-3)$

×		

$(5x-2)$

$(5x+2)$

×		

Expanding 2

1. Expand and simplify
   a. $(2x+3)(x-2)$
2. Expand and simplify
   i. $3x(x+3)+4\,(x+3)$
3. Expand and simplify
   i. $(x+6)^2+(x-3)^2$
4. Expand and simplify $\left(2-\sqrt{3}\right)^2$
5. Simplify $\dfrac{2}{(x+3)}+\dfrac{x-3}{x}$
6. Expand and simplify $(x^3-7)(x^3+7)$
7. Expand and simplify
   1. $(3x+2)(4x^2+2x-3)$
8. Simplify $\dfrac{2x-2}{(x+2)}-\dfrac{x-2}{3x}$

Prove It!

Write some digits in a circle. E.g.

5
3 4
8 6

- The sum of the squares of the two-digit numbers read clockwise is:
  $54^2 + 46^2 + 68^2 + 83^2 + 35^2 = 17770$
- The sum of the squares of the two-digit numbers read anti-clockwise is:
  $53^2 + 38^2 + 86^2 + 64^2 + 45^2 = 17770$

Prove that the two sums will always be equal for any circle of digits.

Advanced Mathematics Support Programme®

More Brackets

Did you know?

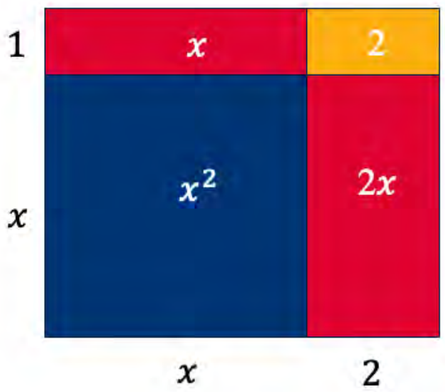
Have a think about the expression (x + 2)(x + 1)

Formal Method

$$(x+2)\,(x+1)$$
$$= x\,(x+1) + 2(x+1)$$
$$= x^2 + x + 2x + 2$$

$$= x^2 + 3x + 2$$

Geometrical Representation



$$= x^2 + 3x + 2$$

Grid Method

	x	$+2$
$+1$	x	2
x	x^2	$2x$

$$= x^2 + 3x + 2$$

- This expression expands to give 4 terms…which simplify to 3 terms.
- How many terms are in the un-simplified expansion of $(x+3)(x+4)(x+5)$?
- Be prepared to explain your thinking…

The story so far….

1. Expand and simplify
$$(2x+3)(3x-5)$$

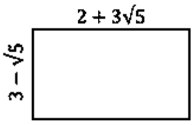
2. Write $(x+3)^2-4$ in the form ax^2+bx+c

3. Expand and simplify
$$(2a+2)(3x-4a+3)$$

4. Expand and simplify
$$3x(x-3)(x+5)$$

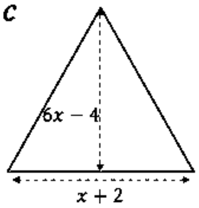
5. Evaluate (no calc allowed)
$$\left(2+\frac{1}{3}\right)\left(2-\frac{1}{3}\right)$$

6. Find the area of this rectangle



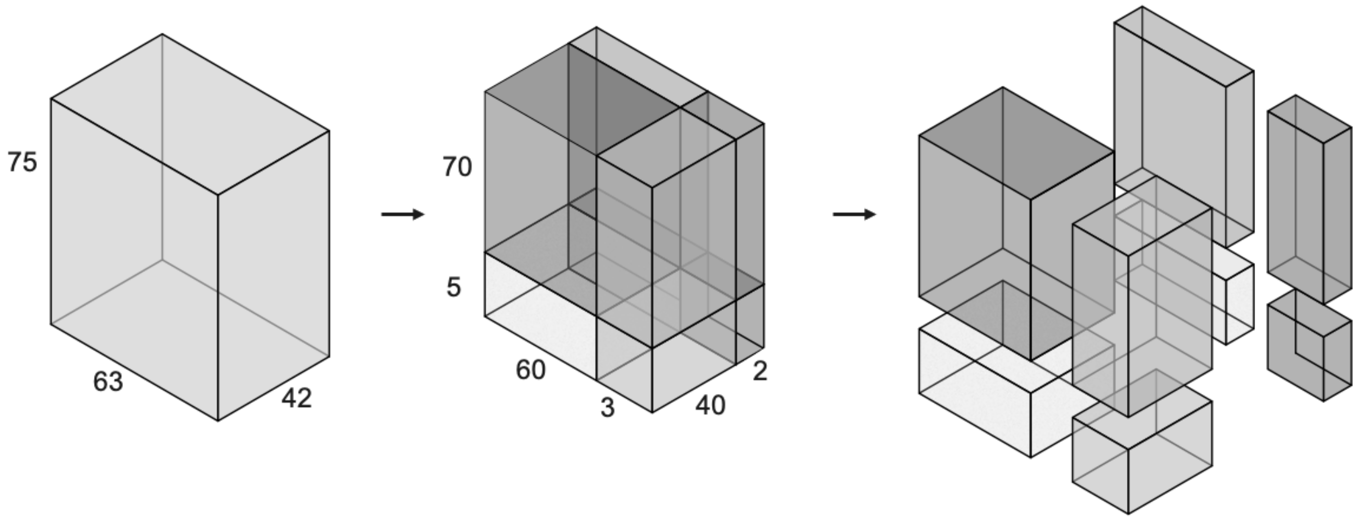
7. Expand and simplify
$$(5-4x)(3x+6)+(5x-2)(3+4x)$$

8. Find the area of the triangle and write it in the form ax^2+bx+c



Have a look…

These diagrams represent a method for calculating 63 x 42 x 75



- Can you see what is going on?
- What are the values of the coloured sections?
- This diagram should help you answer the question: “How many terms there are in an un-simplified expression $(x+3)(x+4)(x+5)$?”

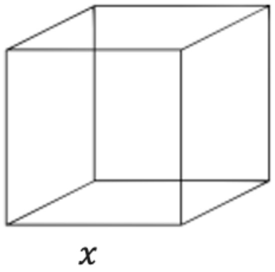
Think again

How might you go about expanding the following?

$$(x + 2)(x + 1)(x + 2)$$

- Is there a way you could use some of the previous workings?
- How would the geometric diagram have to change?
- Could you use a grid method to speed things up?

Getting Bigger



Here is a cube with side lengths of x cm

The cube is going to have its lengths increased in one of three ways

Method A	Method B	Method C
Each side is increased by 2 units	One side is increased by 3 units, one side is increased by 2 units, and one side is increased by 1 unit	One side is increased by 5 units, one side is increased by 2 units, and one side is decreased by 1 unit

- Can you prove which of the solids will have the largest volume?

Expanding Cubics and Beyond

Previously we saw how we could use a grid for expanding brackets such as (1 + x)^2

	1	$+x$
1	1	$+x$
$+x$	$+x$	$+x^2$

So $(1+x)^2 = 1 + 2x + 1x^2$

	1	$+2x$	$+x^2$
1	1	$+2x$	$+x^2$
$+x$	$+x$	$+2x$	$+x^3$

So $(1+x)^3 = 1 + 3x + 3x^2 + 1x^3$

Can you use a similar approach to expand:

- $(1+x)^4$
- $(1+x)^5$
- $(1+x)^6$

Pascal's Triangle 1

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

- Look carefully at this triangle of numbers – have you seen it before?
- Can you work out any patterns that are present?
- Take a careful look at the coefficients that you have found in the previous task.
- Can you see a connection?

Pascal's Triangle 2

Here's the same triangle, referenced with the coefficients of a (1 + x)^n expansion

Row 0: $(1+x)^0 = 1$ → 1

Row 1: $(1+x)^1 = 1 + x$ → 1 1

1 2 1

Row 3: $(1+x)^3 = 1 + 3x + 3x^2 + 1x^3$ → 1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

→ 1 6 15 20 15 6 1

Row 6: $(1+x)^6 = 1 + 6x + 15x^2 + 20x^3 + 15x^4 + 6x^5 + 1x^6$

Use the triangle to help you complete these expansions:

$(\mathbf{1}+\boldsymbol{x})^{\mathbf{7}}$ $(\mathbf{1}+\boldsymbol{x})^{\mathbf{8}}$

Summary and review

1. Expand and simplify
$\left(\frac{1}{3}x + \frac{1}{9}\right)\left(3x - \frac{2}{3}\right)$

2. Expand and simplify
$(x+1)(x+2)(x+3)$

3. Expand and simplify
$(x-3)(x+2)^2$

4. Expand and simplify
$(2 - \sqrt{3})(1+\sqrt{3})(1-\sqrt{3})$

5. Find the volume of a cube with side length
$x - 4$

6. Expand and simplify
$(x^2-2)(x^2+2)(x+1)$

7. Write $\left(\sqrt{y}+\sqrt{8y}\right)^2$ in the form $a + b\sqrt{2}$.

Given that $\left(\sqrt{y}+\sqrt{8y}\right)^2 = 54 + b\sqrt{2}$.
Find values for y and b.

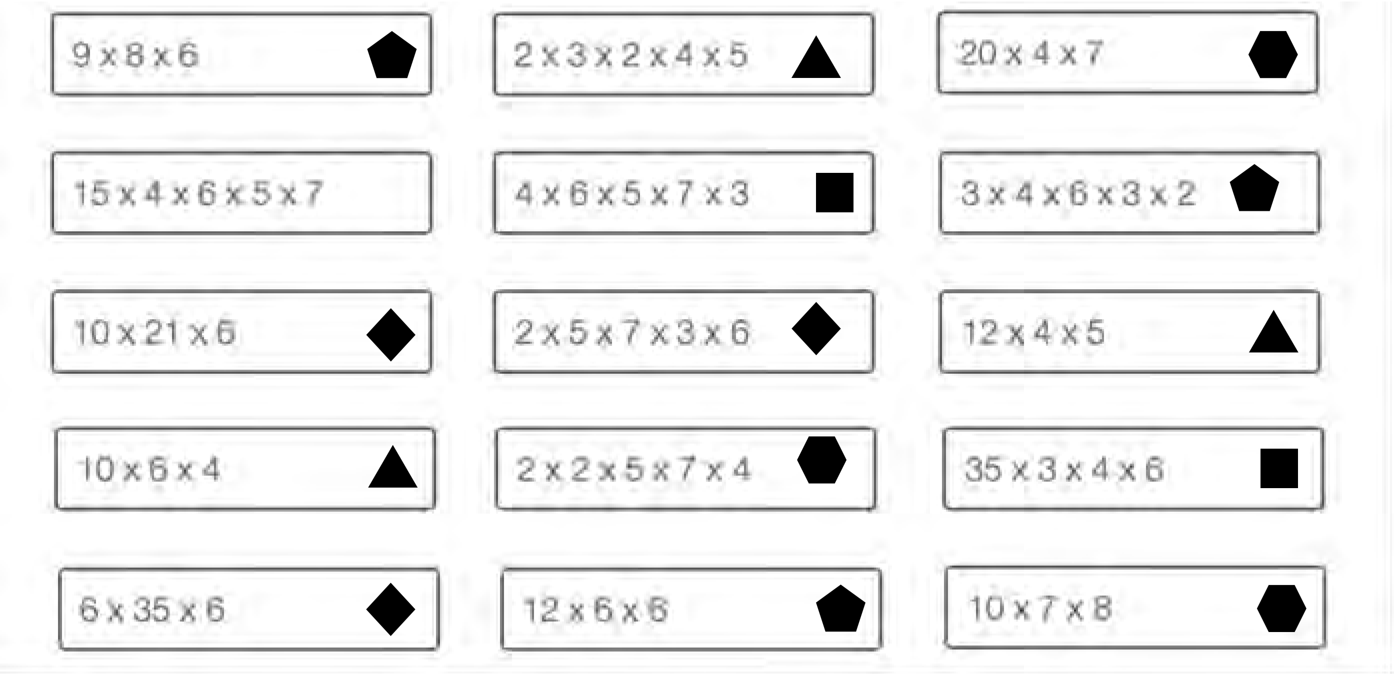
8. Simplify $\frac{(x-1)(x+2)}{(x+3)} - \frac{4}{2x+1}$

Expanding Solutions
Expanding

Expanding 1

1. 3 x 14 & (3 x 4) + (3 x 10)
2. 12 x 2 x 14 & 3 x 7 x 8 x 2 & 2 x 7 x 6 x 4
3. $4x + 20$
4. $2\frac{1}{6}$
5. 2 x 4 x 2 x 6
6. $2(6x + 3)$ & $6(2x + 1)$
7. $-6y - 3x$
8. $9x - 8$

Multiplication Matching



9 x 8 x 6 ⬟	2 x 3 x 2 x 4 x 5 ▲	20 x 4 x 7 ⬢
15 x 4 x 6 x 5 x 7	4 x 6 x 5 x 7 x 3 ■	3 x 4 x 6 x 3 x 2 ⬟
10 x 21 x 6 ◆	2 x 5 x 7 x 3 x 6 ◆	12 x 4 x 5 ▲
10 x 6 x 4 ▲	2 x 2 x 5 x 7 x 4 ⬢	35 x 3 x 4 x 6 ■
6 x 35 x 6 ◆	12 x 6 x 6 ⬟	10 x 7 x 8 ⬢

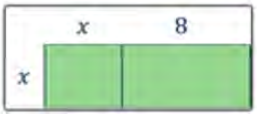
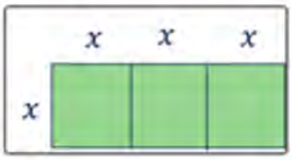
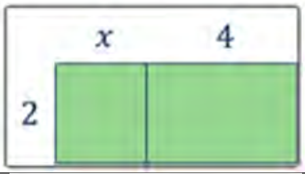
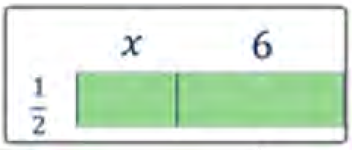
Did you find the odd one out?

Geometric Interpretations

Total	A	B	C	D	Total
81	9 x 9	9(3 + 6)	(9 x 3) + (9 x 6)	27 + 54	81
60	5 x 12	5 (4 + 8)	(5 x 4) + (5 x 8)	20 + 40	60
72	9 x 8	9 (3 + 5)	(9 x 3) + (9 x 5)	27 + 45	72
90	15 x 6	15 (4 + 2)	(15 x 4) + (15 x 2)	60 + 30	90
144	12 x 12	12 (+ 5)	(12 x 7) + (12 x 5)	84 + 60	144
36	3 x 12	3 (8 + 4)	(3 x 8) + (3 x 4)	24 + 12	36

Expanding Solutions

Match it up!

Add eight to x, then multiply by x		$x(x+8)$
Add twelve to x, then divide by three		$\frac{1}{3}(x+12)$
Square x, then multiply by three		$3x^2$
Add four to x then, multiply by two		$2x+8$
Add five to x then. multiply by x		$x(x+5)$
Halve x, then add three		$\frac{x+6}{2}$

Odd one out

$3(2x + 3y) - (x - y)$

$x(x + 4) + 6(x + 3)$

$2(x + 4) + x(x + 4)$

$x(x - 6) + 2(x - 6)$

Expanding 2

1. $2y^2 - 3y$
2. $6x^3y - 4x^5$
3. $11x - 5$
4. $6\sqrt{2} - 8$
5. $y(y + 4)$ & $y^2 + 4y$
6. $6(2x + 8)$ or $(12x + 48)$ cm^2
7. $3a - 4t - 2$
8. $\frac{7a^2-52a}{24}$

Expanding Solutions

Double Brackets

Dealing with Negativity

2. -168
2. -162
3. 24
4. - 40
5. 7a^2
6. 18ab^2
7. 168a^3
8. With an EVEN number of negative numbers then value will be positive.

   With an ODD number of negative numbers then value will be negative.

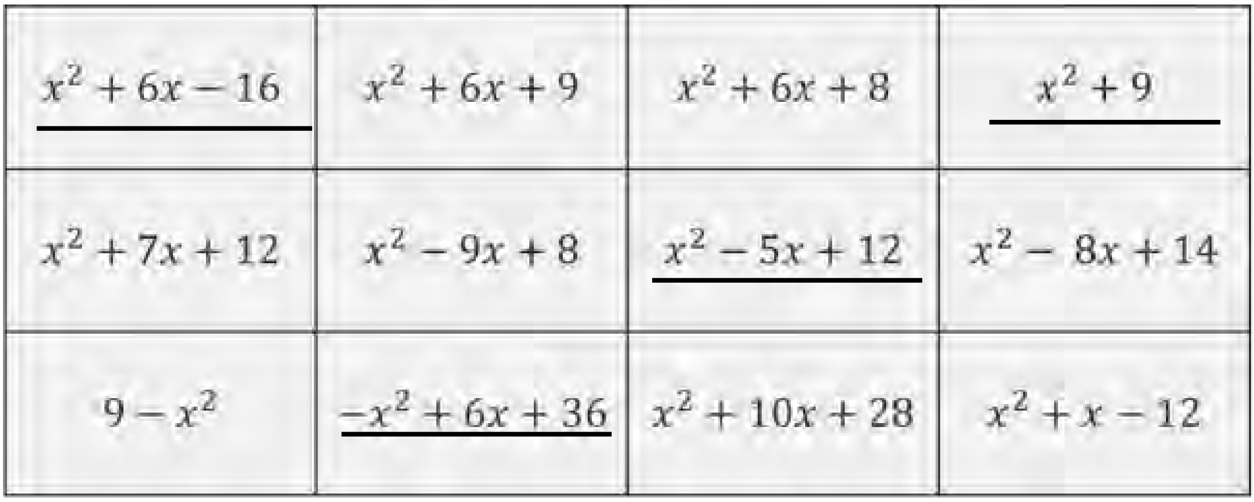
Expanding 1

1. Negative because there are an odd number of negative numbers
2. 2 x 3 x 17 x 2 x 2, (20 + 4) (10 + 7), 20(10 + 7) + 4(10 + 7)
3. $3\sqrt{3} - 18$
4. $x^2 + 7x + 10$
5. $x^2 + 4x - 12$
6. $4\sqrt{2} + 5$
7. $x^4 + 8x^2 + 12$
8. $x^2 + 3x^3 + 7x^2 + 21$

What's gone wrong?

1. Correct
2. $x^2 + 5x + 6$
3. $x^2 - x - 20$
4. -7
5. $\frac{2x^2 + 3x + 12}{4x}$
6. $x^2 + 4x + 4$

Expand and Simplify



$\underline{x^2 + 6x - 16}$	$x^2 + 6x + 9$	$x^2 + 6x + 8$	$\underline{x^2 + 9}$
$x^2 + 7x + 12$	$x^2 - 9x + 8$	$\underline{x^2 - 5x + 12}$	$x^2 - 8x + 14$
$9 - x^2$	$\underline{-x^2 + 6x + 36}$	$x^2 + 10x + 28$	$x^2 + x - 12$

The four expressions left to simplify to $2x^2 + 7x + 41$

Expanding Solutions

Quadratic Puzzles

1. $x^2 - 8x + 16$
2. $2x^2 + 5x + 3$
3. $3x^2 + 4x - 15$
4. $4x^2 - 2x - 12$
5. $6x^2 - x - 12$
6. $25x^2 - 4$

Note that the **product** of the diagonals give identical results (this might be useful to know at a later point).

Expanding 2

1. $2x^2 - x - 6$
2. $3x^2 + 13x + 12$
3. $2x^2 + 6x + 45$
4. $7 - 4\sqrt{3}$
5. $\frac{x^2+2x-9}{x(x+3)}$
6. $x^6 - 49$
7. $12x^3 + 14x^2 - 5x - 6$
8. $\frac{5x^2-6x+4}{3x(x+2)}$

More Brackets

The story so far

2\. $6x^2 - x - 15$

2\. $x^2 + 6x + 5$

3\. $6ax - 8a^2 - 2a + 6x + 6$

5\. $3x^3 - 6x^2 - 45x$

5\. $3\frac{8}{9}$

6\. $7\sqrt{5} - 9$

7\. $8x^2 - 2x + 24$

8\. $3x^2 + 4x - 4$

Have a look…

There will be 8 terms in the un-simplified expression $x^3 + 2x^2 + 3x^2 + 6x + 2x + 4$

They simplify to become $x^3 + 5x^2 + 8x + 4$

Getting Bigger…

The following explains the solution

A. $(x+2)(x+2)(x+2)$

$$= (x+2)(x^2+4x+4)$$
$$= x^3 + 4x^2 + 4x + 2x^2 + 8x + 8$$
$$= x^3 + 6x^2 + 12x + 8$$

B. $(x+3)(x+2)(x+1)$

$$= (x+3)(x^2+3x+2)$$
$$= x^3 + 3x^2 + 2x + 3x^2 + 9x + 6$$
$$= x^3 + 6x^2 + 11x + 8$$

C. $(x+5)(x-1)(x+2)$

$$= (x+5)(x^2+x-2)$$
$$= x^3 + x^2 - 2x + 5x^2 + 5x - 10$$
$$= x^3 + 6x^2 + 3x - 10$$

Because x is a side length we know that x is positive. Therefore A is the greatest as $12x + 8$ is larger than $11x + 8$ and $3x - 10$.

Expanding Solutions

Expanding Cubic Solutions

Note the pattern in the co-efficients which are in **bold**

$$(x+1)^4 = \mathbf{1} + \mathbf{4}x + \mathbf{6}x^2 + \mathbf{4}x^3 + \mathbf{1}x^4$$

$$(1+x)^5 = \mathbf{1} + \mathbf{5}x + \mathbf{10}x^2 + \mathbf{10}x^3 + \mathbf{5}x^4 + \mathbf{1}x^5$$

$$(1+x)^6 = \mathbf{1} + \mathbf{6}x + \mathbf{15}x^2 + \mathbf{20}x^3 + \mathbf{15}x^4 + \mathbf{6}x^5 + \mathbf{1}x^6$$

Pascal's Triangle

You should see that the pattern in the coefficients in the expansions form Pascal's Triangle.

Summary and review

1. $x^2 + \frac{1}{9}x - \frac{2}{27}$
2. $x^3 + 6x^2 + 11x + 6$
3. $x^3 + x^2 - 8x - 12$
4. $2\sqrt{3} - 4$
5. $x^3 - 12x^2 + 48x - 64$
6. $x^5 + x^4 - 4x - 4$
7. $y = 6 \quad b = 24$
8. $\frac{2x^3+3x^2-7x-14}{(x+3)(2x+1)}$

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Factorising 1

Did you know?

How would you work out
$32 \div 8$?

- Did you use the fact that you know $8 \times 4 = 32$?

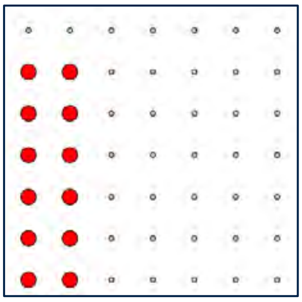
Often, we use multiplication to help us do division as it is more straightforward.

- The same is true for factorising and expanding.
- It can often be easier to expand than to factorise
- So, use expanding to help you factorise

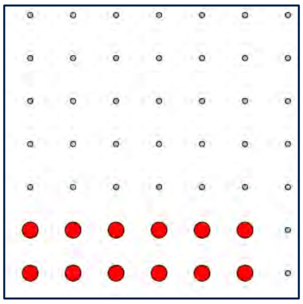
Factors and Rectangles

I have 12 red counters and a large sheet of dotted paper.
How many different rectangular arrays can I make using all 12 counters?

A

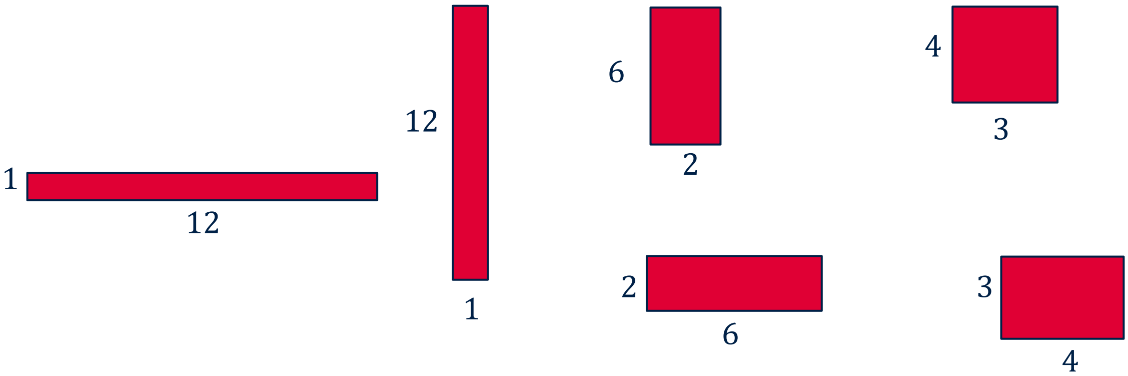


B



- An array is an arrangement of objects in rows and columns
- For this activity we will count A and B as different arrays as they have different orientations

This problem is equivalent to finding the number of rectangles with area 12 that have integer length sides, and counting 2 by 6 as different to 6 by 2



There are six arrays for 12 counters.

Factoring and Rectangles

How many different arrays are there for:

- 7 counters?
- 15 counters?
- 25 counters?
- A prime number of counters?
- What is special about numbers with an odd number of arrays?

Factorising 1

Fully factorise the following:

1. $5x-30$
2. $9x+6$
3. x^2+6x
4. $6y^3-12y$
5. $7a^2b+21ab-14a$
6. $12x^2+12xy+12y^2$
7. $3t(t-1)+7(t-1)$
8. $2x(x^2+3)-5(x^2+3)$

Factorising 2

Fully factorise the following

1. $7x+28$
2. $14-21x$
3. y^2-8y
4. $3t^4+9t^2$
5. $3x^3y-12xy^2+6xy$
6. $8a^3b+6y^2b-10b$
7. $6x(x+3)+5(x+3)$
8. $7y(3-2y)-2(3-2y)$

Enough Information

You are told that

$$ab = 245$$

$$bc = 635$$

$$a + c = 88$$

What is the value of b?

Enough Information Hints

- Try adding the first two expressions together
- Now factorise
- Have another look at the question

Square Root

By considering prime factors, and without a calculator, find the square root of 27×147

Square Root Hints

- Draw prime factor trees for 27 and 147 separately
- Write down 27 x 147 expressed as a product of their prime factors
- Simplify the expression
- Have another look at the question

The Root Cause

Simplify $\sqrt{2y^2(x+3)^2 + 7(x+3)^2y^2}$

The Root Cause Hint

- Factorise first (Q7 and Q8 from Factorising 1 will help)
- Have another look at the question

Power Puzzle

Simplify

$$\frac{4x^{2.5} - 6\sqrt{x}}{2x^2 - 3}$$

Power Puzzle Hint

- Rewrite $\sqrt{x}$ as a power of x
- What is 2.5 as a fraction?
- Factorise the numerator
- Have another look at the question

Factor Problem

Pick 3 different integers from 1 to 10

Place your numbers in the boxes in as many different ways as possible (i.e 6 ways)

$$\square(\square x + \square)$$

- Write down all the expressions
- Multiply them all out
- Add up all your results and simplify
- Now factorise that answer

Try again with a different set of 3 numbers

What do you notice? Can you prove it?

Further Factorising

Did you know?

Substitute $x = 9$ into the following two expressions

$$\boldsymbol{x^2 + 3x + 2}$$

What do you notice?

$$(9)^2 + 3(9) + 2 = 81 + 27 + 2 = 110$$

$$\boldsymbol{(x + 2)(x + 1)}$$

$$(9 + 2)(9 + 1) = 11 \times 10 = 110$$

Both give the same answer as the expressions are equivalent

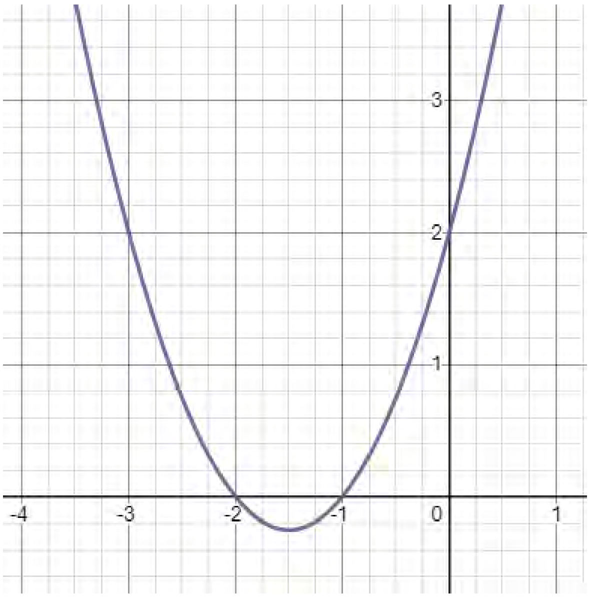
One of the expressions was a lot easier to evaluate! Why?

Which is best?

$x^2 + 3x + 2$ *or* $(x + 2)(x + 1)$

expanded form factorised form

$y = x^2 + 3x + 2$ $y = (x + 2)(x + 1)$



Factorising is a key skill for both sketching graphs and solving equations, both of which will be covered later.

Sometimes it is more helpful to factorise an expression, other times better to be expand it, depending on the context.

Further Factorising 1

Factorise the following fully:

1. $x^2 + 5x - 6$
2. $x^2 + 13x - 30$
3. $y^2 - 13y + 30$
4. $t^2 + 2t - 15$
5. $k^2 - 2k - 24$
6. $p^2 - 10p + 21$
7. $x^2 - 16x$
8. $3x(2x - 1) + 4(1 - 2x)$

Further Factorising 2

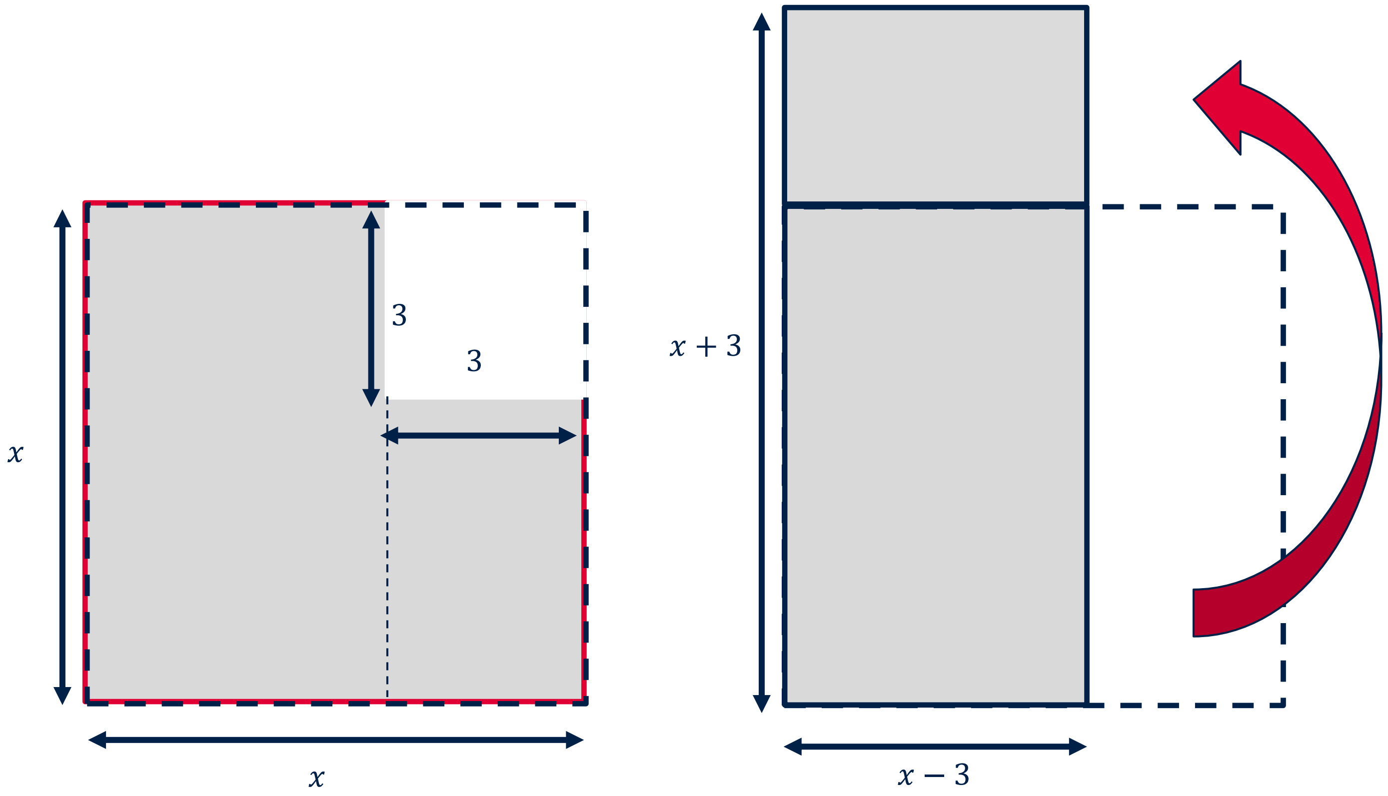
Factorise the following fully:

1. $x^2 + 6x - 7$
2. $y^2 + y - 12$
3. $y^2 - 11y + 28$
4. $t^2 + 7t - 18$
5. $k^2 + 9k + 20$
6. $x^2 + x - 56$
7. $p^2 - 25p$
8. $x^2(3x - 4) + (4 - 3x)$

Difference of Two Squares

A special case for factorising is the difference of two squares.

Expressions such as $x^2 - 3^2$, where the coefficient of x is zero.



$$x^2 - 3^2 \quad = \quad (x-3)(x+3)$$

Try factorising these expressions using the difference of two squares

1. $x^2 - 6^2$
2. $y^2 - 144$
3. $x^2 - y^2$
4. $4t^2 - 81$
5. $x^2 - 5$

$ax^2 + bx + c$

So far we have been factorising quadratic expressions where $a = 1$. For example, $x^2 - 2x - 15$

Time to try some trickier quadratics!

Have a go at this one…

Factorise $\boldsymbol{6x^2 + 19x + 10}$

If you got $6x^2 + 19x + 10 = (3x + 2)(2x + 5)$ Well done!

Feeling confident? You can try the Trickier Quadratics questions below

There are many methods for factorising quadratics where $a > 1$

You might want to refresh your memory on the method that you learnt at school if you are going to tackle the following questions.

Trickier Quadratics

1. $\boldsymbol{3x^2 - 10x - 8}$
2. $\boldsymbol{2x^2 - 7x + 6}$
3. $\boldsymbol{4y^2 + 20y + 9}$
4. $\boldsymbol{6x^2 - 13x - 8}$
5. $\boldsymbol{20x^2 + x - 12}$

Further Factorising Problems

These expressions are slightly different to the previous ones but can still be factorised.

1. $2t^2 - 32$
2. $x^3 - 7x^2 + 12x$
3. $x^4 - x^2 - 2$
4. $y^4 - 625$

Without a calculator

What is the value of each of the following?
calculators not allowed

$$9^2 - 1^2$$

$$99^2 - 1^2$$

$$999^2 - 1^2$$

Still without a calculator

Without using a calculator, find the value of

$$\frac{122 \times (122^2 + 4 \times 123)}{124} - \frac{124 \times (124^2 - 4 \times 123)}{122}$$

Top and Bottom

Simplify

$$\frac{x^2 - 3x - 10}{x^2 + 7x + 10}$$

Some possible hints!

Without a calculator Hint	Still without a calculator Hint	Top and Bottom Hint
■ Can you factorise $9^2 - 1^2$? ■ How does this help?	■ Replace 123 by n and 122 by n−1 ■ Now go on to factorise	■ Factorise the numerator then the denominator ■ What do you notice?

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Completing the Square

Did you know?

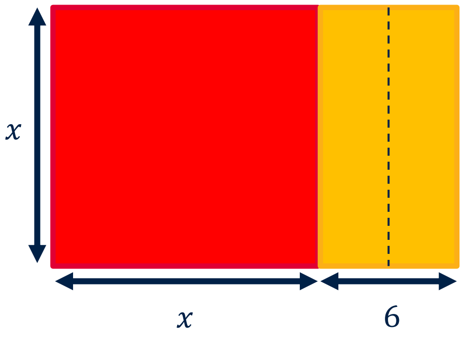
These are different forms of the same algebraic expression

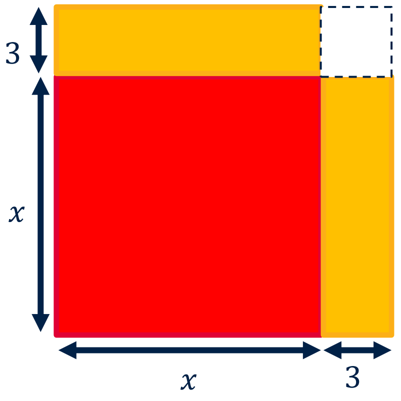
$$x^2 + 6x \quad = \quad x(x+6) \quad = \quad (x+3)^2 - 9$$

expanded form

factorised form

completed square form





Do the diagrams help you see why this is called **Completing the square**?

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Completing the square 1

Write these expressions in the form $(x+a)^2+b$

1. x^2+4x
2. x^2+4x+5
3. y^2-8y
4. y^2-8y+7
5. $x^2-12x+41$
6. $k^2+10k-2$
7. y^2+3y+1
8. p^2-2p+1

Completing the square 2

Write these expressions in the form $(x+a)^2+b$

1. x^2+10x
2. $x^2+10x+30$
3. y^2-2y
4. y^2-2y+3
5. $x^2-8x+25$
6. $k^2+14k-1$
7. y^2+5y+6
8. t^2+6t+9

Different Forms

It is important to be able to convert expressions between the different forms:

expanded form   factorised form   completed square form

In this problem there are 4 sets of three equivalent expressions, however, some expressions are missing. Match the sets and find the 3 missing expressions.

$a^2 - 2a - 8$		$a^2 - 8a + 15$
	$a^2 + 2a - 15$	$(a + 2)(a + 4)$
$(a + 1)^2 - 16$	$(a - 3)(a - 5)$	
$(a + 5)(a - 3)$	$(a - 1)^2 - 9$	$(a + 3)^2 - 1$

Extra Puzzle 1

What is the value of

$$\frac{\frac{(5^2 - 3^2)}{5 + 3} + \frac{(4^2 - 2^2)}{4 + 2} + \frac{(3^2 - 1^2)}{3 + 1}}{2} \; ?$$

Extra Puzzle 2

Given that

$$55^2 - 45^2 = (55 + 45)(55 - 45) = 1000$$

and

$$60^2 - 40^2 = (60 + 40)(60 - 40) = 2000$$

- Find numbers a and b such that $a^2 - b^2 = 3000$
- Find numbers c and d such that $c^2 - d^2 = 4000$
- Find numbers e and f such that $e^2 - f^2 = 100\,000$

Factorising Solutions

Factorising

Factorising 1

1. $5(x-6)$
2. $3(3x+2)$
3. $x(x+6)$
4. $6y(y^2-2)$
5. $7a(ab+3b-2)$
6. $12(x^2+xy+y^2)$
7. $(t-1)(3t+7)$
8. $(x^2+3)(2x-5)$

Fractions 2

1. $7(x+4)$
2. $7(2-3x)$
3. $y(y-8)$
4. $3t^2(t^2+3)$
5. $3xy(x^2-4y-2)$
6. $2b(4a^3+3y^2-5)$
7. $(x+3)(6x+5)$
8. $(3-2y)(7y-2)$

Enough Information

$\boldsymbol{ab+bc=245+635 \rightarrow b(a+c)=880 \text{ so } b=10}$

Square Root

63

The Root Cause

$\boldsymbol{3y(x+3)}$

Power Puzzle

$\boldsymbol{2\sqrt{x}}$

Factor Problem

$Choosing\ 3, 5\ and\ 8\ gives$

$$\begin{aligned}
\boldsymbol{3(5x+8)} &\boldsymbol{= 15x+24}\\
\boldsymbol{3(8x+5)} &\boldsymbol{= 24x+15}\\
\boldsymbol{5(3x+8)} &\boldsymbol{= 15x+40}\\
\boldsymbol{5(8x+3)} &\boldsymbol{= 40x+15}\\
\boldsymbol{8(3x+5)} &\boldsymbol{= 24x+40}\\
\boldsymbol{8(5x+3)} &\boldsymbol{= 40x+24}
\end{aligned}$$

$\boldsymbol{Total\ is}$: $158x+158\ or\ 158(x+1)$

$So\ x+1\ is\ factor\ for\ every\ final\ expression$

Further Factorising

Further Factorising 1

1. $(x+6)(x-1)$
2. $(x+15)(x-2)$
3. $(y-10)(y-3)$
4. $(t+5)(t-3)$
5. $(k-6)(k+4)$
6. $(p-7)(p-3)$
7. $x(x-16)$
8. $(2x-1)(3x-4)$

Factorising Solutions

Further Factorising 2

1. $(x+7)(x-1)$
2. $(y+4)(y-3)$
3. $(y-7)(y-4)$
4. $(t+9)(t-2)$
5. $(k+5)(k+4)$
6. $(x+8)(x-7)$
7. $p(p-25)$
8. $(3x-4)(x^2-1)$

Difference of Two Squares

1. $(x-6)(x+6)$
2. $(y+12)(y-12))$
3. $(x+y)(x-y)$
4. $(2t-9)(2t+9)$
5. $(x+\sqrt{5})(x-\sqrt{5})$

Trickier Quadratics

1. $(3x+2)(x-4)$
2. $(2x-3)(x-2)$
3. $(2y+1)(2y+9)$
4. $(3x-8)(2x+1)$
5. $(5x+4)(4x-3)$

Further Factorising Problems

1. $2(t-4)(t+4)$
2. $x(x-3)(x-4)$
3. $(x^2-2)(x^2+1)$
4. $(y^2+25)(y-5)(y+5)$

Without a calculator 80, 9800, 998 000

Still without a calculator 0

Top and Bottom $\frac{x-5}{x+5}$

Completing the Square

Completing the square 1

1. $(x+2)^2-4$
2. $(x+2)^2+1$
3. $(y-4)^2-16$
4. $(y-4)^2-9$
5. $(x-6)^2+5$
6. $(k+5)^2-27$
7. $\left(y+\frac{3}{2}\right)^2-\frac{5}{4}$
8. $(p-1)^2$

Factorising Solutions

Completing the square 2

1. $(x+5)^2-25$
2. $(x+5)^2+5$
3. $(y-1)^2-1$
4. $(y-1)^2+2$
5. $(x-4)^2+9$
6. $(k+7)^2-50$
7. $\left(y+\frac{5}{2}\right)^2-\frac{1}{4}$
8. $(t+3)^2$

Different Forms Solution

Each **column** is the same quadratic but in different forms

$\boldsymbol{a^2+6a+8}$	$\boldsymbol{(a-4)^2-1}$	$\boldsymbol{(a-4)(a+2)}$	$\boldsymbol{a^2+2a-15}$
$\boldsymbol{(a+2)(a+4)}$	$\boldsymbol{(a-3)(a-5)}$	$\boldsymbol{a^2-2a-8}$	$\boldsymbol{(a+1)^2-16}$
$\boldsymbol{(a+3)^2-1}$	$\boldsymbol{a^2-8a+15}$	$\boldsymbol{(a-1)^2-9}$	$\boldsymbol{(a+5)(a-3)}$

Extra Puzzles $\boldsymbol{\frac{2+2+2}{2}=3}$

One possible solution………

$$\boldsymbol{a^2-b^2=(65+35)(65-35)}$$

$$\boldsymbol{c^2-d^2=(70+30)(70-30)}$$

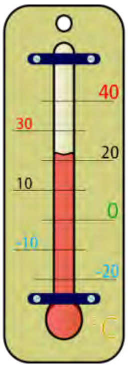
$$\boldsymbol{e^2-f^2=(550+450)(550-450)}$$

Rearranging

Did you know?

This is a well-known formula that you might recognise.

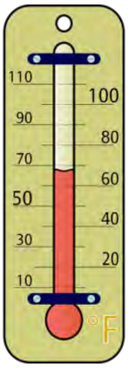
$$F = \frac{9}{5}C + 32$$



It is used to change temperatures in degrees

Celsius °C to degrees Fahrenheit °F

$$°C \longrightarrow °F$$



For example: If it is 20°C to find the temperature in °F

you simply substitute C=20 into the formula above:

What would I need to do if I wanted to convert from Fahrenheit to Celsius??

$$°C \longleftarrow °F$$

Rearranging 1

1. Solve $3x + 25 = 60$
2. Rearrange $z = w + 3$ to make w the subject
3. Rearrange $5x - 4 = 2y$ to make x the subject
4. Rearrange $y = \frac{t}{6}$ to make t the subject
5. $y = 6p^2 + 2$ rearrange to make p the subject
6. The area of a circle is found using $\text{A} = \pi r^2$ Write the equation you would use to find the radius.
7. In a right angled triangle $sinx = \frac{Opp}{Hyp}$ write down the equation for finding the opposite side.
8. To change temperatures in Celsius to Fahrenheit this formula is used.

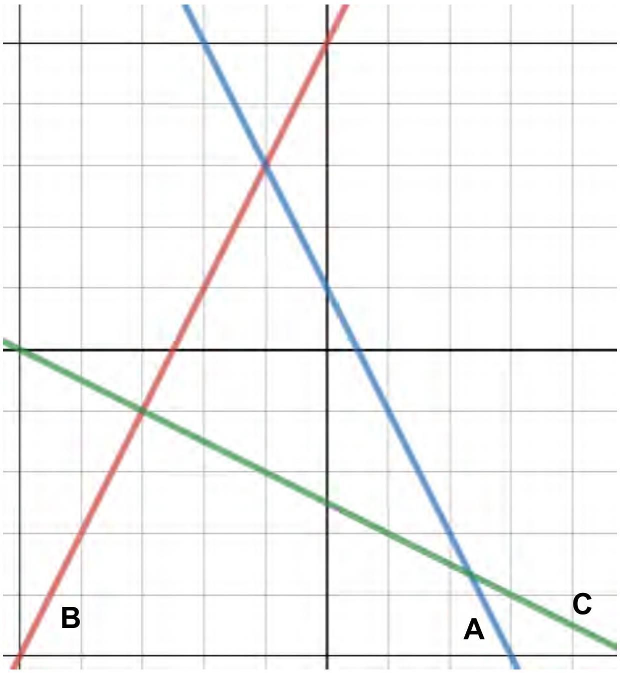
$$F = \frac{9}{5}C + 32$$

Rearrange to give the formula for converting Celsius to Fahrenheit

Linear Graphs 2

1. Make x the subject of $x - f = y + b$
2. Make y the subject $ty - x^2 = b$
3. Make c the subject $ac + d = m^2$
4. Make a the subject $x(a - e) = d$
5. Make y the subject $b(y - b) = b^2$
6. To find velocity, v, we use the formula
$$v^2 = u^2 - 2as$$
Rearrange to find s
7. The area of a sector of a circle is given by
$$A = \frac{\theta\pi r^2}{360}$$
Express θ in terms of A, π and r
8. Make x the subject $m(y - x) = t$

Line them up 1

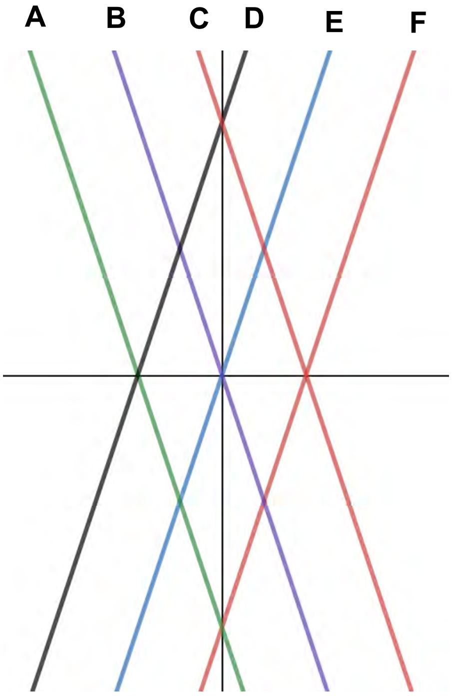


Which is which?

- $y = 2x + 5$
- $2y + x + 5 = 0$
- $y + 2x = 1$

How does rearranging enable you to justify your answer?

Line them up 2



- $y = 4 - 3x$
- $y + 3x + 4 = 0$
- $y + 3x = 0$
- $y = 3x$
- $y = 3x + 4$
- $y - 3x + 4 = 0$

Pairing up

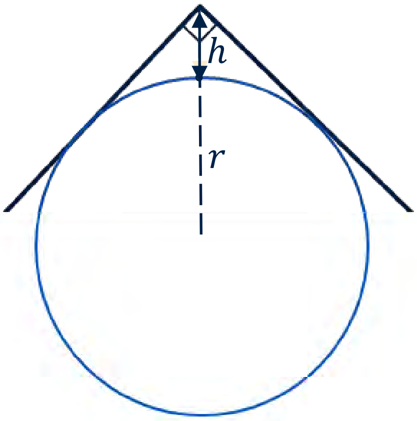
Can you sort the cards into pairs under the following headings:

1. These lines are perpendicular
2. These lines have the same x intercept
3. These lines have the same y intercept
4. These lines are parallel
5. These lines go through the point (1,5)
6. These lines …

$3y = 2x - 8$	$y = -(x + 8)$	$y = 4x + 4$
$2y + x = 4$	$y = 6x - 4$	$y = 8x - 3$
$y + x + 8 = 0$	$2y = 8x + 3$	$4y = x + 3$
$2y + 8 = 3x$	$y + 6x = 11$	$y + 4x + 6 = 0$

Pipe Problem

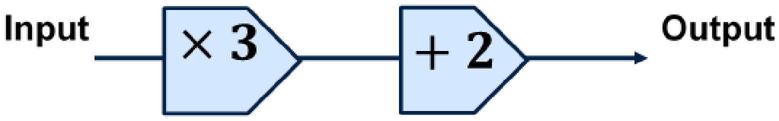
Can you find the radius of the pipe shown if the only measurement you can take is the one marked h?



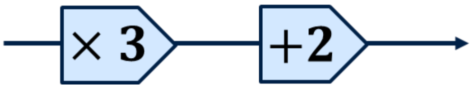
Rearranging and Functions

A function relates an input to an output

Here is an example of a function machine



Complete the following table for the function machine shown



Input	**Output**
5	
-4	
x	
	17
	x

What do you notice?

Rearranging and Functions Solutions

Let's introduce function notation that you will use in A level maths:

If $f(x) = 3x + 2$ then to find the inverse function we do the reverse

so we subtract 2 then divide by 3

This gives us the inverse function which we call $f^{-1}(x)$

In this case $\boldsymbol{f^{-1}(x) = \frac{x-2}{3}}$

Important! The inverse should give us back the original value

Let's check: $\boldsymbol{f(5) = 17 \; and \; f^{-1}(17) = 5}$

Rearranging and Functions

Original function

$f(x) = 3x + 2$

Inverse function

$f^{-1}(x) = \frac{x-2}{3}$

Find the inverse of each of these functions.

1. $f(x) = 3x - 5$
2. $f(x) = 4x + 7$
3. $f(x) = \dfrac{x}{2} + 1$
4. $f(x) = \dfrac{x+2}{3}$
5. $f(x) = \dfrac{2}{3}x + 3$
6. $f(x) = 3 - 2x$

Instead of reversing a function machine - try re-arranging the original function to make x the subject

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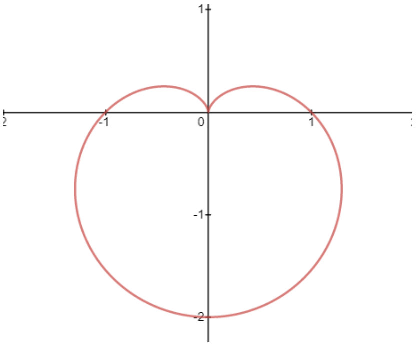
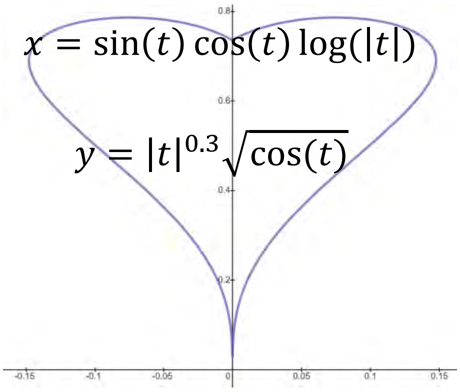
Rearranging Factorising

Did you know?

- Being able to express equations in different forms gives us different information
- Later we'll be looking at information needed to sketch graphs
- If you continue your maths studies to A Level Further Maths, you will draw graphs such as these

$$r = 1 - sin\theta$$

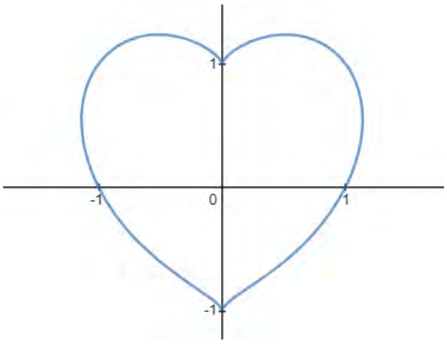




$$x = \sin(t)\cos(t)\log(|t|)$$

$$y = |t|^{0.3}\sqrt{\cos(t)}$$

$$(x^2 + y^2 - 1)^3 - x^2y^3 = 0$$



Futher Factorising 1

1. The equation of a line is given as
    i. $3y + 4x \ -2 = 0.$
    b. What is the gradient of the line?
2. A rectangle has area A, length y and width $x - 2$. Write an expression for the length of the rectangle, y, in terms of A and x
3. Make x the subject of:
    a. $ax - y = z + bx$
4. The equation of a line is given as
    i. $5(\mathrm{b} - \mathrm{p}) = 2(\mathrm{b} + 3)$
5. John says the first step to rearranging
    a. $\frac{x-a}{f} = 3g$ is to add $a \ to \ 3g$. Is he right? Explain your answer.
6. Make a the subject of
    a. $5(a \ - t) = 3(a + x)$
7. Make x the subject of
    a. $ay + x = 4x + xb$
8. Make x the subject of
    a. $2\pi\sqrt{x + t} = 4$

Further Factorising 2

1. Make y the subject of
   $xy + 6 = 7 \ - ky$
2. Find an expression for the area of a rectangle with length, $(y - x)$ and width, $(x - 2)$
3. Rewrite your expression in Q2 to have y expressed in terms of A and x
4. Make y the subject of
   $\frac{4}{y} + 1 = 2x$
5. Displacement can be expressed as
    i. $s = ut + \frac{1}{2}at^2$

   Express a in terms of s, u and t
6. Make y the subject of $\sqrt{by^2 - x} = D$
7. The area of a trapezium has formula
    i. $A = \frac{1}{2}\left(\frac{a+b}{h}\right)$

   Express h in terms of $A,\ a$ and b
8. Make t the subject $b(t + a) = x(t + b)$

Equivalent quadratics

Sort the expressions below in to 4 sets of 4 equivalent expressions

$x^2 - 25$	$2x^2 - 2$
$(x+5)(x+6) - x - 55$	$(x+5)(x-5)$
$2(x^2 - 1)$	$(x+5)^2 - 10x - 50$
$2(x+3)(x-1)$	$2(x+1)(x-1)$
$(x+5)^2 - 50$	$2(x+2)^2 - 4x - 14$
$2x^2 + 4x - 6$	$(x+5)(x-5) + 10x$
$2(x+1)^2 - 8$	$(x-5)(x+6) - x + 5$
$x^2 + 10x - 25$	$2(x+1)^2 - 4(x+1)$

Mean squares

- Take two positive values greater than 1
- Find the mean of the two values
- Square it

THEN

- Take the same two values
- Square them
- Find the mean of the squares

Which value is greater?

Is this always true?

Can you prove it?

Hint

- Try out several examples
- Is one expression always bigger than the other?
- Next try using x and y instead.
- If you subtract one expression from the other, can you work out if it's positive or negative?

Difference of numeric squares

Problem 1

Mrs Gryce was asked to calculate 18×12 by Mr Lo who had forgotten his calculator and was doing some marking.

Mrs Gryce quickly responded

"Well, that's just $15^2 - 9$ which is 216"

Mr Lo was amazed.

- **How did she know so quickly what the answer was?**

Problem 2

Use the fact that 3 x 4 = 12

Can you quickly work out a value for $(3.5)^2$?

- **Can you see a connection between the previous question and this one?**

The Quadratic Formula

We've all used the Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- But where does it come from?
- Can you prove why the quadratic formula works?

Rearrange these steps in order to prove the quadratic formula

$$ax^2 + bx + c = 0 \longrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\left(x + \frac{b}{2a}\right) = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} = -\frac{c}{a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$ax^2 + bx = -c$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right) = \pm\sqrt{\left(\frac{b^2 - 4ac}{4a^2}\right)}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

Match the steps below with the algebra above for a slightly easier version

Step 1: Subtract c from both sides

Step 2: Divide both sides by a

Step 3: Complete the square on the left hand side

Step 4: Add $\frac{b^2}{4a^2}$ to both sides

Step 5: Make the right hand side into a single expression

Step 6: Take the square root of both sides

Step 7: Simplify the denominator on the right hand side

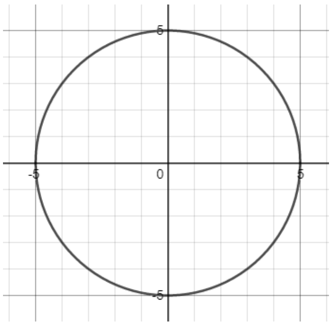
Step 8: Subtract $\frac{b}{2a}$ from both sides

Step 9: You now have the quadratic formula!

Equations of Circles

$$x^2 + y^2 = 25$$

Represents a circle with centre (0,0) and radius 5



Generally, the equation of a circle with centre (0,0) and radius r can be written as

$$\boldsymbol{x^2 + y^2 = r^2}$$

- **What happens if the centre is not (0,0)?**

Let's have a look at this equation: $x^2 + 4x + y^2 - 6y = 12$

We can rearrange this by completing the square separately for the x terms and y terms

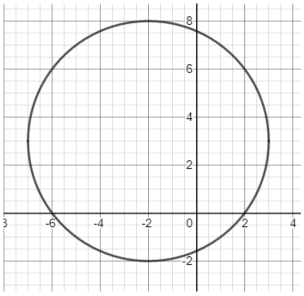
$$x^2 + 4x = (x+2)^2 - 4 \text{ and } y^2 - 6y = (y-3)^2 - 9$$

So $x^2 + 4x + y^2 - 6y = 12$

Can be written as $(x+2)^2 - 4 + (y-3)^2 - 9 = 12$

$$(x+2)^2 + (y-3)^2 - 13 = 12$$

$$(x+2)^2 + (y-3)^2 = 25$$



$$(x+2)^2 + (y-3)^2 = 25$$

Represents a circle with Centre (-2,3) and radius 5

- Can you find the centre and radii of these circles by rearranging into the form

$$(x+a)^2 + (y-b)^2 = r^2$$

$x^2 - 8x + y^2 - 2y = 19$

$x^2 + 6x + y^2 - 10y = 15$

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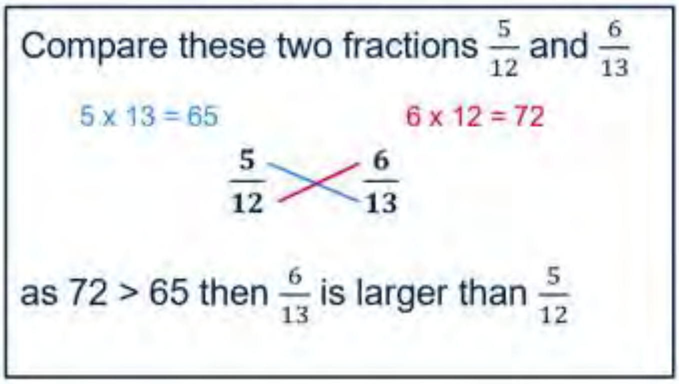
Rearranging Fractions

Did you know?

Comparing Fractions

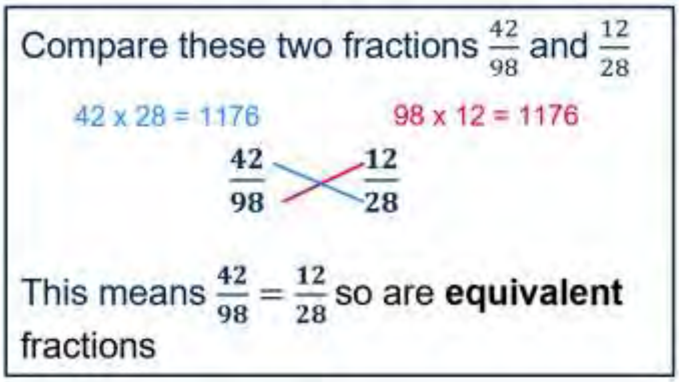
To order fractions you can compare the product of their diagonals



Compare these two fractions $\frac{5}{12}$ and $\frac{6}{13}$

$5 \times 13 = 65$ $6 \times 12 = 72$

as $72 > 65$ then $\frac{6}{13}$ is larger than $\frac{5}{12}$



Compare these two fractions $\frac{42}{98}$ and $\frac{12}{28}$

$42 \times 28 = 1176$ $98 \times 12 = 1176$

This means $\frac{42}{98} = \frac{12}{28}$ so are **equivalent** fractions

If fractions are equivalent then the product of their diagonals will always be equal!

How could you use this to help you when rearranging or solving equations involving fractions?

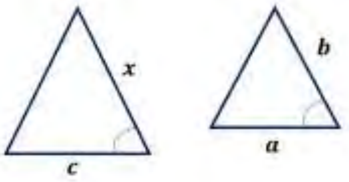
Fractions 1

1. Rewrite the formula to make time the subject
$$Speed = \frac{distance}{time}$$

2. Rearrange to make a the subject
$$\frac{x}{y} = \frac{a}{b}$$

3. Make x the subject of $tan\theta = \frac{y}{x}$

4. These triangles are similar.
Show that $x = \frac{cb}{a}$



5. Make x the subject of $x = \frac{h+k}{a}$

6. Make x the subject of $x + a = \frac{x+b}{c}$

7. Make a the subject of $\frac{1-a}{1+a} = \frac{x}{y}$

8. Make y the subject of $y(\sqrt{3} + \sqrt{2}) = x$
And write in the form $y = x(\sqrt{a} + \sqrt{b})$

Fractions 2

1. Make x the subject of $bc = \frac{x}{a}$

2. Make e the subject of $x = \frac{y}{e^2}$

3. Write a in terms of x, y, z and b.
$$\frac{b-xa}{z} = y$$

4. Make v the subject $C = \frac{v^2-ta}{x}$

5. Rearrange to make x the subject of
$$\frac{2}{x} + 5 = 6y$$

6. Make x the subject of
$$4F = F + \frac{a}{y+x}$$

7. Make y the subject of
$$\sqrt{\frac{m(y+a)}{y}} = y$$

8. A cylinder has a radius 3 cm and height h cm. The total surface area is $30x\ cm^2$
Find an expression for surface area and write h in terms of x and π

Wrong Steps

Each expression has been written in different ways

- Which are not correct rearrangements?
- Can you explain what's gone wrong?

	$c = \frac{3e^2}{d}$
A.	$d = 3e^2 - c$
B.	$cd = 3e^2$
C.	$\frac{d}{e^2} = \frac{c}{3}$
D.	$\frac{1}{3}c = \frac{e^2}{d}$
E.	$d = \frac{3e^2}{c}$

	$\frac{sinx}{4} = \frac{siny}{a}$
A.	$\frac{a}{4} = \frac{siny}{sinx}$
B.	$siny = \frac{4}{asinx}$
C.	$sinx = \frac{4siny}{a}$
D.	$asinx = 4siny$
E.	$a = \frac{sinx}{4siny}$

	$\frac{T-a}{T+a} = \frac{x}{y}$
A.	$x(T+a) = y(T-a)$
B.	$xy - ay = yT - ya$
C.	$a = \frac{y(T-a)}{x+y}$
D.	$xa + ya = yT - xT$
E.	$a = \frac{x+y}{yT-ya}$

	$a - \frac{b^2}{d} = ce$
A.	$b^2 = d(a+ce)$
B.	$a = ce + \frac{b^2}{d}$
C.	$\frac{b^2}{d} = a - ce$
D.	$\frac{b}{\sqrt{d}} = \sqrt{a} - \sqrt{ce}$
E.	$b = \pm\sqrt{d(a-ce)}$

	$y + b = \frac{ay+e}{b}$
A.	$by + b^2 = ay + e$
B.	$by - ay = e + b^2$
C.	$y = \frac{e-b^2}{b-a}$
D.	$e = b(y+b) - ay$
E.	$y(b-a) = \frac{e-b^2}{y}$

Prove it

Using your rearranging skills can you prove each of the following

If $$a = \frac{b}{b+c}$$

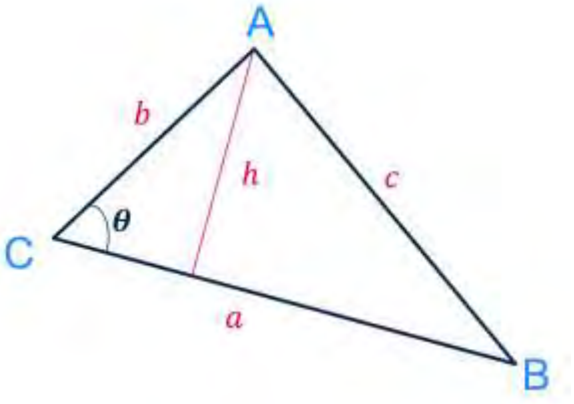
Show that $$\frac{a}{1-a} = \frac{b}{c}$$

$\frac{n(n-1)}{2} + \frac{n(n+1)}{2}$ is a square number

$$\frac{2x+3}{4} - \frac{3x\ -2}{3} + \frac{1}{6} = \frac{19-6x}{12}$$

Missing Steps

Complete the steps and fill in the blanks to find an expression for the area of triangle ABC



1. On the diagram draw a perpendicular line from A to BC
2. Label the perpendicular line, h h
3. Find an expression for the perpendicular height, h

$h =$

Hint: you might want to use some trigonometry here

4. Write down the expression for the base of the triangle

$base =$

5. Write down an expression to find the area of this triangle using your expressions for $base$ and $perpendicular\ height$

$area =$

Rearranging Solutions

Rearranging

Rearranging 1

1. $x = \frac{35}{3}$
2. $w = z - 3$
3. $x = \frac{2y+4}{5}$
4. $t = 6y$
5. $p = \pm\sqrt{\frac{y-2}{6}}$
6. $r = \sqrt{\frac{A}{\pi}}$
7. $Opp = Hyp \times sinx$
8. $\frac{5}{9}(F - 32) = C$

Rearranging 2

1. $x = y + b + f$
2. $y = \frac{(b+x^2)}{t}$
3. $c = \frac{m^2-d}{a}$
4. $a = \frac{d}{x} + e$
5. $y = 2b$
6. $s = \frac{v^2-u^2}{2a}$
7. $\theta = \frac{360A}{\pi r^2}$
8. $x = y - \frac{t}{m}$

Please note that there may be alternative correct expressions – check with your teacher

Line them up 1

$A\ (y + 2x = 1)$ $\quad$ $B\ (y = 2x + 5)$ $\quad$ $C\ (2y + x + 5 = 0)$

Line them up 2

$A\ (y + 3x + 4 = 0)$ $\quad$ $B\ (y + 3x = 0)$ $\quad$ $C\ (y = 4 - 3x)$

$D\ (y = 3x + 4)$ $\quad$ $E\ (y = 3x)$ $\quad$ $F\ (y - 3x + 4 = 0)$

Pairing up

Perpendicular lines

$4y = x + 3$

$y + 4x + 6 = 0$

Parallel Lines

$y = 4x + 4$

$2y = 8x + 3$

Same x-intercept

$2y + x = 4$

$3y = 2x - 8$

Same y-intercept

$2y + 8 = 3x$

$y = 6x - 4$

Go through (1,5)

$y = 8x - 3$

$y + 6x = 11$

These are..the same line

$y + x + 8 = 0$

$y = -(x + 8)$

Pipe Problem $r = \frac{h}{\sqrt{2}-1}$

Rearranging Solutions

Rearranging Functions -

1. $f^{-1}(x) = \frac{x+5}{3}$

2. $f^{-1}(x) = \frac{x-7}{4}$

3. $f^{-1}(x) = 2(x-1)$

4. $f^{-1}(x) = 3x-2$

5. $f^{-1}(x) = \frac{3(x-3)}{2}$

6. $f^{-1}(x) = \frac{3-x}{2}$

Rearranging Factorising

Rearranging Factorising 1

1. $-\left(\frac{4}{3}\right)$	2. $y = \frac{A}{x-2}$	3. $x = \frac{z+y}{a-b}$	4. $b = \frac{6+5p}{5-2x}$
5. $No, \times$ $by\ f\ first$	6. $a = \frac{3x+5t}{2}$	7. $x = \frac{ay}{3+b}$	8. $x = \frac{4}{\pi^2} - t$

Rearranging Factorising 2

1. $y = \frac{1}{x+k}$	2. $A = xy - x^2 - 2y + 2x$	3. $y = \frac{2x-x^2-A}{(2-x)}$	4. $y = \frac{4}{2x-1}$
5. $\frac{2s-2ut}{t^2}$	6. $y = \pm\sqrt{\frac{D^2+x}{b}}$	7. $h = \frac{a+b}{2A}$	8. $t = \frac{xb-ba}{b-x}$

Equivalent Quadratics

$x^2 - 25$ $(x+5)(x-5)$ $(x+5)^2 - 10x - 50$ $(x+5)(x+6) - x + 5$	$2x^2 - 2$ $2(x^2 - 1)$ $2(x+1)(x-1)$ $2(x+1)^2 - 4(x+4)$	$2(x+3)(x-1)$ $2(x+2)^2 - 4x - 14$ $2x^2 + 4x - 6$ $2(x+1)^2 - 8$	$(x+5)(x+6) - x - 55$ $(x+5)^2 - 50$ $(x+5)(x-5) + 10x$ $x^2 + 10x - 25$

Rearranging Solutions

Mean Squares

Mean of two positive numbers then squared $\frac{x^2+y^2+2xy}{4}$

Mean of squares $\frac{x^2+y^2}{2}$

Difference between means $\frac{(x-y)^2}{4}$ *which must be positive as the numerator is squared*

Difference of numeric squares

Problem 1 $18\ x\ 12$ *can be written as* $(15+3)(15-3) = 15^2 - 3^2 = 216$

Problem 2 - $3\ \times 4 = 12$ *or* $(3.5-0.5)(3.5+0.5) = 12$ *so* $3.5^2 = 12 + 0.5^2$ *which is* 12.25

Quadratic Formula

ask your teacher for the full worked solution to this

Equations of Circles

$(x-4)^2 + (y-1)^2 = 36$ *circle with centre* $(4, 1)$ *and radius* 6

$(x+3)^2 + (x-5)^2 = 42$ *circle with centre* $(-3, 5)$ *and radius* 7

Rearranging Fractions

Rearranging Fractions 1

1. $time = \frac{distance}{speed}$
2. $a = \frac{xb}{y}$
3. $x = \frac{y}{tan\theta}$
4. $x = \frac{bc}{a}$
5. $a = \frac{h+k}{x}$
6. $x = \frac{b-ca}{c-1}$
7. $a = \frac{y-x}{x+y}$
8. $y = x(\sqrt{3} - \sqrt{2})$

Rearranging Fractions 2

1. $abc = x$
2. $e = \sqrt{\frac{y}{x}}$
3. $a = \frac{b-zy}{x}$
4. $v = \pm\sqrt{Cx + ta}$
5. $x = \frac{2}{6y-5}$
6. $x = \frac{a-3FY}{3F}$
7. $y = \frac{ma}{g^2-m}$
8. $h = \frac{(5x-3\pi)}{\pi}$

Rearranging Solutions

Wrong Steps

For each expression the wrong steps are indicated underneath

$c=\frac{3e^2}{d}$	$\frac{sinx}{4}=\frac{siny}{a}$	$\frac{T-a}{T+a}=\frac{x}{y}$	$a-\frac{b^2}{d}=ce$	$y+b=\frac{ay+e}{b}$
$A\ and\ C$	$B\ and\ E$	$B\ and\ E$	$A\ and\ D$	$B\ and\ E$

Can you prove it

$a=\frac{b}{b+c}$ $\frac{a}{1}=\frac{b}{b+c}$ $a(b+c)=b$ $ac=b-ab$ $b=\frac{ac}{1-a}$ $\frac{b}{c}=\frac{a}{1-a}$	$\frac{n(n-1)}{2}+\frac{n(n+1)}{2}$ $\frac{n^2-n+n^2+n}{2}$ $\frac{2n^2}{2}$ n^2 $therefore\ the\ starting\ is\ square$	$\frac{2x+3}{4}-\frac{3x-2}{3}+\frac{1}{6}$ $\frac{3(2x+3)}{12}-\frac{4(3x-2)}{12}+\frac{2}{12}$ $\frac{6x+9-12x+8+2}{12}$ $\frac{19-6x}{12}$

Missing Steps

$sin\theta=\frac{h}{b}\ so\ h=b\ sin\theta$

$Base=a$

$Area=\frac{1}{2}base\times perpendicular\ height\ \ which\ becomes\ \ A=\frac{1}{2}a\times bsin\theta\ \ or\frac{1}{2}absin\theta$

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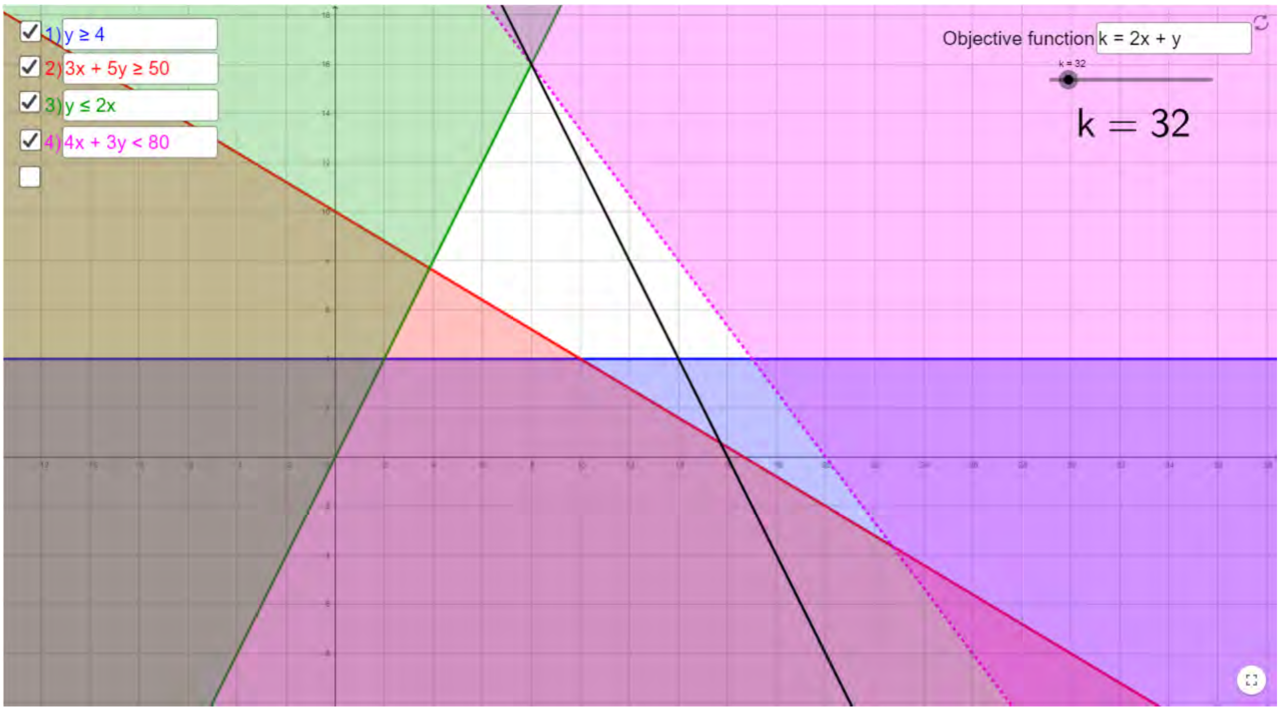
Solving Linear Equations

Did you know ?

Linear programming is a method that involves solving a set of linear equations or inequalities in order to find the best solution.



☑ 1) $y \geq 4$
☑ 2) $3x + 5y \geq 50$
☑ 3) $y \leq 2x$
☑ 4) $4x + 3y < 80$
☐

Objective function $k = 2x + y$

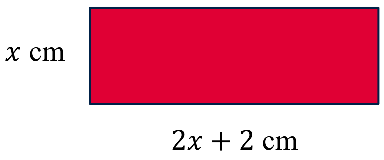
$k = 32$

It is very useful in industry for finding the best level of production, or the maximum profit depending on varying costs, sales, mix of products or availability of labour etc…

Solving Linear 1

Solve the equations

1. $8x - 3 = 5x + 13$
2. $3x + 1 > 10$ and $2x + 7 < 15$
3. $3(x + 6) > 12$
4. $24 - 3x = 9$
5. $14 \geq 8 + 5x$
6. $6 - 2x < 5x + 34$
7. $\frac{2x+3}{7} = \frac{4x-5}{3}$
8. The perimeter of the rectangle is 24cm. Find the value of x.



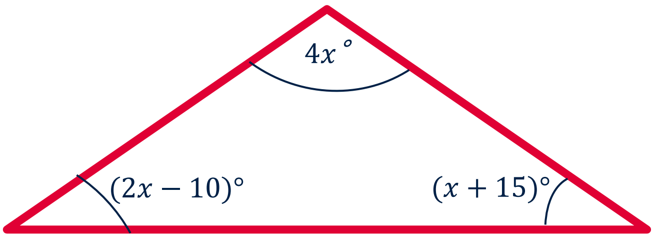
Solving Linear 2

Solve the equations

1. $6x + 5 = 47$
2. $5x + 7 = x + 25$
3. $7(x - 4) = 14$
4. $29 - 4x < 22$
5. $3x < 2x - 1 < 4x + 2$

Hint: Split into two inequalities

6. $19 + 2x = 3x + 15$
7. $\frac{3x-1}{5} \geq \frac{3x+5}{2}$
8. Find the value of x in the triangle below



Piggy in the Middle

The number in the middle of each group of 3 adjoining cells is the average of its two neighbours.

5			**23**	

- What number goes in the right-hand cell?

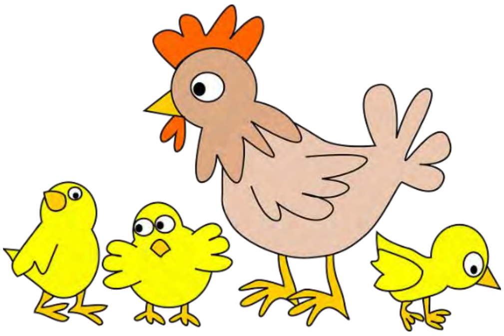
Chicken Run

Victoria has just bought some chickens. She wants to keep them safe in a small enclosure.

The enclosure will be a rectangle where the length is 3m longer than the width.

Victoria has only got 30m of fencing. The area of the enclosure has to be greater than 20m^2.
The length and width are integers.

- How many different size enclosures can Victoria make?



Crack the code

Can you decode this message?

12 1 4 7 5 3 12 4 2 5

7 4 3 3 6 15 4 9 2 6 9 8 4 10

Solve the equations in the boxes below. Each letter will have a different positive integer solution between 0 and 16.

1. $$\frac{4r}{d-4}+\frac{2h}{s}=2$$

2. $$\frac{g-9}{y+4}=\frac{2}{3}$$

3. $$3rh+m=13$$

4. $$\frac{4g}{5}=12$$

5. $$\frac{2c-5+3(c-2)}{2c-1}=2$$

6. $$e^3<72$$

7. $$\frac{s+3y}{8s}=\frac{3}{4}$$

8. $$\frac{6k}{s}-5=11$$

9. $$100<t^2<169$$

10. $$\frac{8}{3a}=\frac{4}{a+3}$$

11. $$\frac{6r+8}{y}=4$$

12. $$2(3m+4)=7m+1$$

Hint:

Try solving the equations in the following order: 4, 5, 10, 2, 11, 12, 7, 8, 3, 1, 9, 6

Linear Simultaneous Equations

There are two main ways to solve simultaneous equations.

Elimination

$$3x + 2y = 9$$

$$5x - 2y = -1$$

Add the two equations together to eliminate y

$$8x = 8$$

$$x = 1$$

Now we have a value for x we can put it into one of the original equations to find y

$$3 \times 1 + 2y = 9$$

$$3 + 2y = 9$$

$$2y = 6$$

$$y = 3$$

Substitution

$$y + 3x = 5$$

$$2y + 7x = 11$$

Rearrange the first equation in terms of y and then substitute into the second equation

$$2(5 - 3x) + 7x = 11$$

$$10 - 6x + 7x = 11$$

$$x = 1$$

Now we have a value for x we can put it into one of the original equations to find y

$$y + 3 \times 1 = 5$$

$$y + 3 = 5$$

$$y = 2$$

Which method is best and when?

Solve the following:

1.

$$2x + y = 7$$

$$2x - y = 1$$

2.

$$3x + 2y = 7$$

$$3x + 5y = 4$$

Which method

will you use

and why?

3.

$$y = 4x + 3$$

$$3x + 2y = 28$$

4.

$$4x + 3y = -4$$

$$6x - 2y = 7$$

Maths at the Movies

Maths movie makes millions!

"Our latest movie 'Sum-body loves you' has generated £15 million in online sales and rentals in the first week of it being released"

Simultaneous Studios said at the weekend.

We are unable to tell you how much of that total represents the £6 digital rental versus the £15 cost of purchasing the movie. But we do know there were 1 945 000 transactions overall.

Is there really no way of finding out how many rentals and how many sales there were of the film?

Use what you have learnt so far to calculate how many individual rentals and sales there were of 'Sum-body loves you'

Taxi!

There are two taxi companies

Initial Charge: £x then £1 per mile	Initial Charge: £$2x$ then 80p per mile

They both charge £12 for a journey of the same distance.

- What is the distance?
- What is the value of x?

Solving Graphically

Use the graphs to solve these pairs of equations

1. $3x + y = 10$

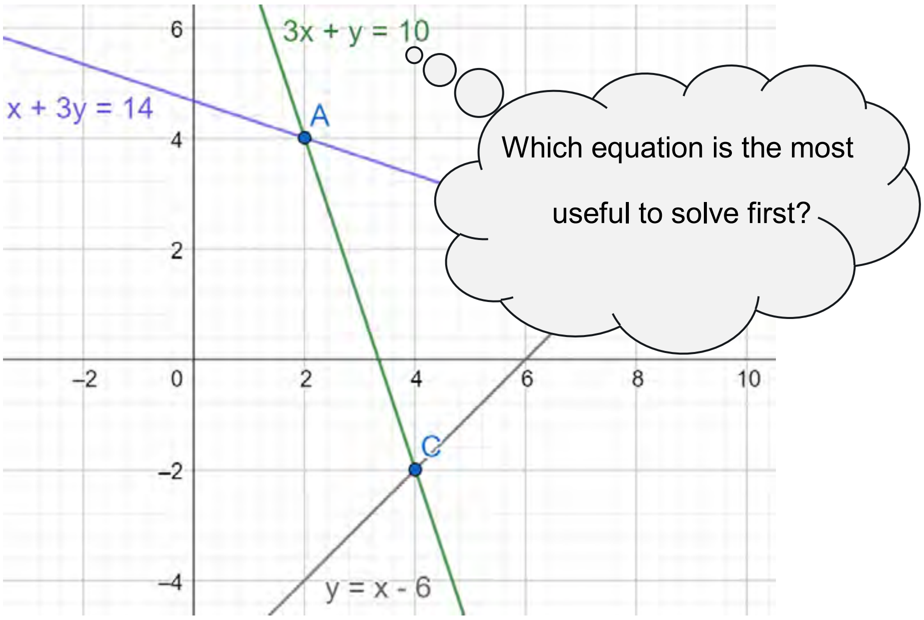
$x + 3y = 14$

2. $y = x - 6$

$3x + y = 10$

3. $x + 3y = 14$

$y = x - 6$



Which equation is the most useful to solve first?

Puzzle to Ponder

Can you explain algebraically why there are no solutions to the simultaneous equations

$$y = 2x + 7$$

$$2y - 4x = 16$$

Triple Simultaneous Equations

Solve:

$$5x + 3y + z = 24$$

$$4y + 2z = 16$$

$$3z = 18$$

Mean Problem

x, y and z satisfy

$$x + y + 3z = 121$$

$$x + 3y + z = 678$$

$$3x + y + z = 356$$

Find the mean of x, y, z, without using a calculator

Hint:

- Write an expression for the mean of x, y, z
- Do you need to find x, y, z seperately to find the mean?

Solving Quadratics

Did you know?

I have picked two numbers that multiply to make zero.

What can you say about my numbers?

At least one of them must be zero

This is useful when using factorising to solve equations.

If $a \times b = 0$, then either $a = 0$ or $b = 0$ (or both!)

Historically zero wasn't accepted as a number until fairly recently!

Solving with Quadratics 1

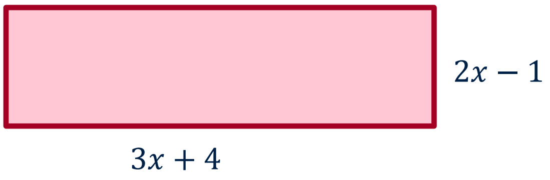
Solve the following

1. $x^2 = 16$
2. $x^2 - 16x = 0$
3. $(x+1)(2x-3) = 0$
4. $x^2 - 3x + 2 = 0$
5. $(2x-5)(4x+3) = 0$
6. $3x^2 + 14x - 5 = 0$
7. $(x+3)^2 = 25$
8. $\frac{3}{x} + \frac{4}{x-1} = 10$

Solving with Quadratics 2

Solve the following

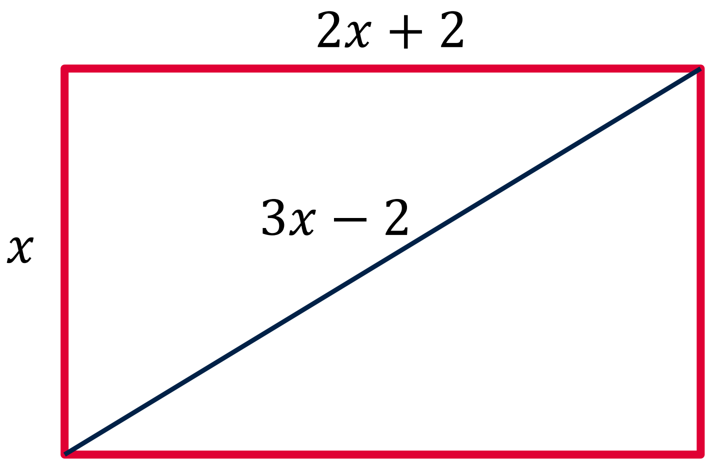
1. $x^2 - 4x - 12 = 0$
2. $x^2 - x = 6$
3. $2x^2 - 11x + 12 = 0$
4. $6x^2 + x - 12 = 0$
5. $3 + 2x - x^2 = 0$
6. $x^2 - 4x - 1 = 0$
7. $\frac{8}{x+2} - \frac{14}{x-3} = 9$
8. The area of this rectangle is $30m^2$



a) Show that $6x^2 + 5x - 34 = 0$
b) Find any possible values for x

Quadthagoras

Find the length, width and diagonal of this rectangle



Up in the air!

An object is launched from a cliff that is $58.8m$ high.

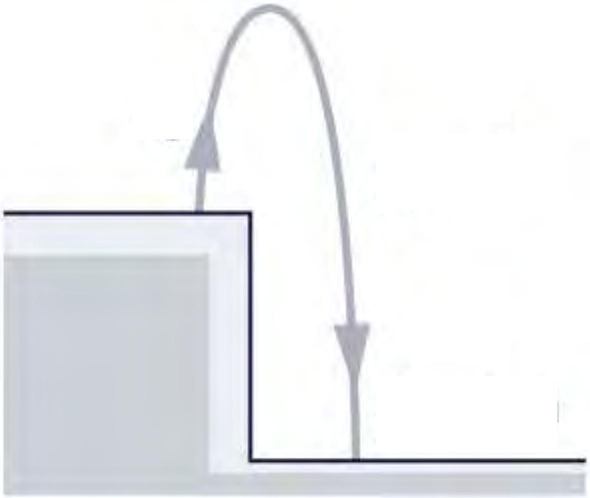
The speed of the object is 19.6 metres per second (m/s).

The equation for the object's height *h* above the ground at time *t* seconds after launch is

$$h \ = \ -4.9t2 \ + \ 19.6t \ + \ 58.8$$

where *h* is in metres.

- When does the object strike the ground?



Which Way?

In the skills check you saw how we can solve quadratic equations by factorising or completing the square.

We can also use the quadratic formula. For a quadratic $ax^2 + bx + c = 0$ the solutions are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Try solving $x^2 + 4x - 21 = 0$ using each of the three methods.

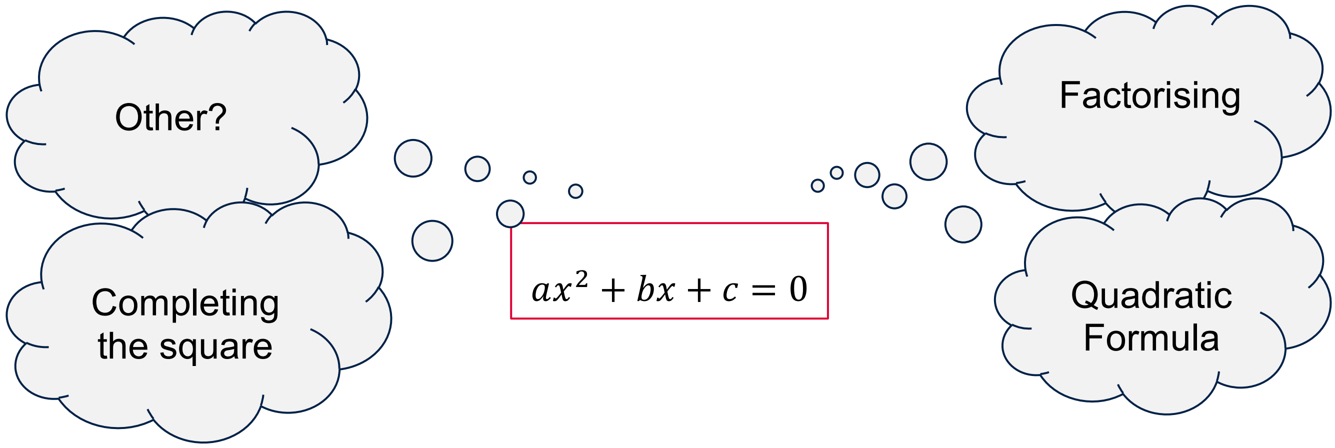
Try solving $3x^2 + 4x - 2 = 0$ using each of the three methods.

Which Way Now?

There is not always one best way to solve a quadratic.

Some methods are better than others for different equations

How can you spot which is the right method for each equation?

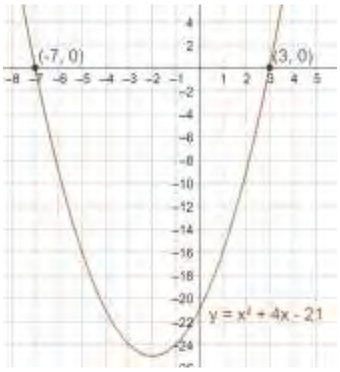


https://undergroundmathematics.org/quadratics/quad-solving-sorter is a really good activity for improving your skills in sorting quadratic equations. You or your teacher may be able to print the cards out to help.

Another Way?

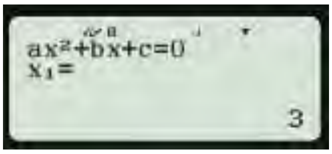
And of course, there are the methods of solving using graphs and/or your calculator

$$x^2 + 4x - 21 = 0$$



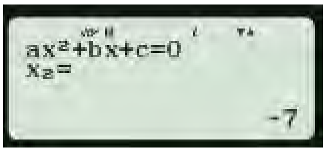
1x²+ 4x 21

-21



ax²+bx+c=0
x₁=

3



ax²+bx+c=0
x₂=

-7

Using Graphs

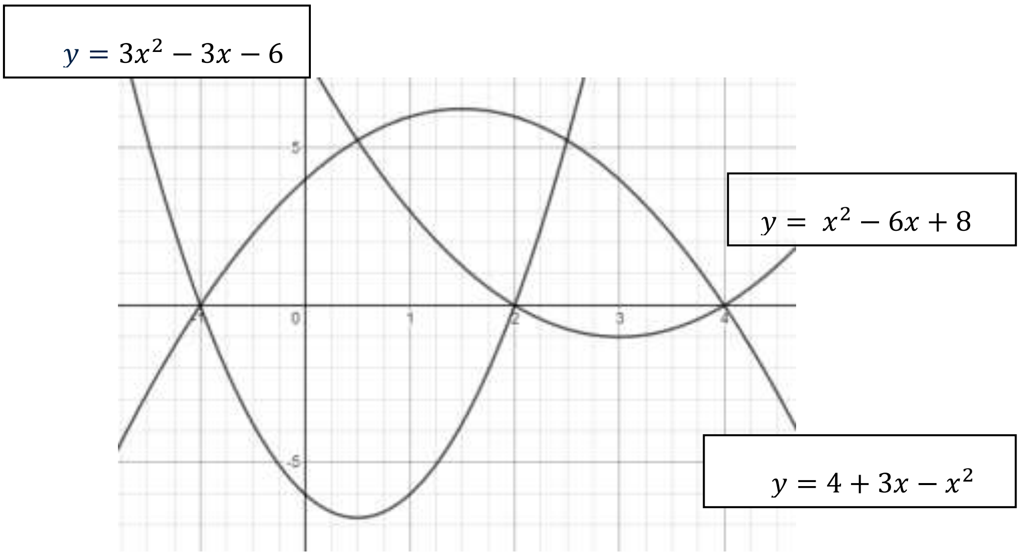
Use the graphs to solve

$$4 + 3x - x^2 = 0$$

$$x^2 - 6x + 8 = 0$$

$$3x^2 - 3x - 6 = 0$$

$$4 + 3x - x^2 = 4$$



Simultaneously

Solve these pairs of equations

1. $y = x^2 + 6x - 9$
$y = 3x + 1$

2. $y = x^2 + 2x + 2$
$y - 4x = 1$

3. A rectangle has length $(a + b)$ and width $3a$.
The area is $60cm^2$ and perimeter is $32\ cm$.
Calculate, algebraically, the values of a and b.

4. In how many places does the line $y = 2x + 2$ intersect the circle $(x + 2)^2 + y^2 = 25$?
What are the co-ordinates of these intersections?

Lines and Curves

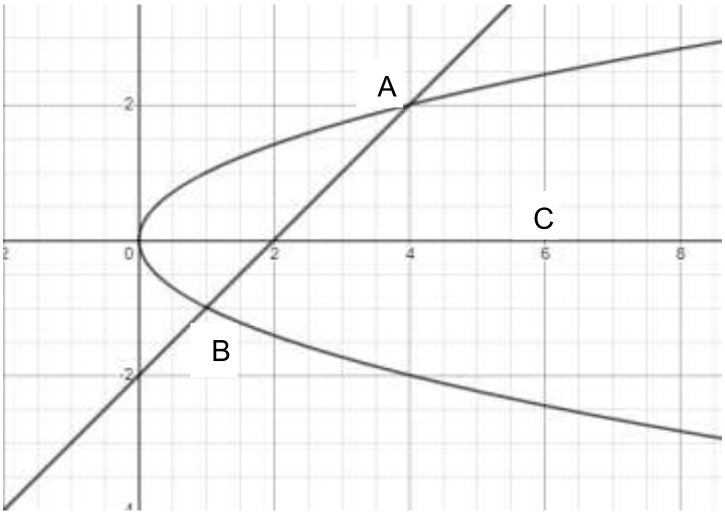
The diagram shows the graphs of

$y^2 = x$ and $y = x - 2$

The graphs cross at the points A and B.

The point C has co-ordinates (6,0)

- Without the use of a calculator, find the exact area of triangle ABC



Solving Other Equations

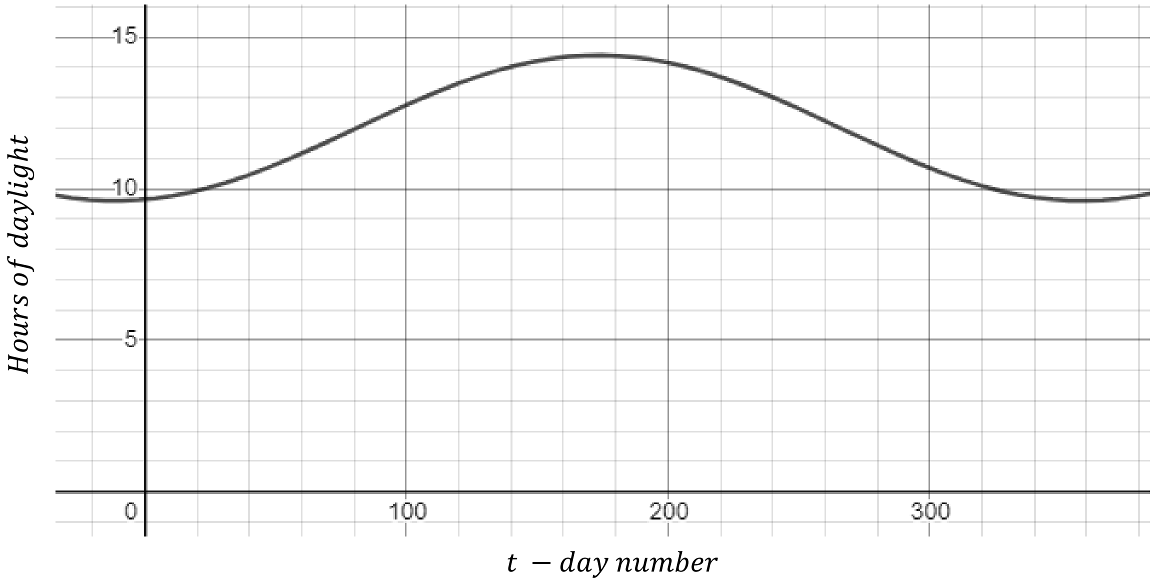
Did you know?

Sunrise and sunset times are modelled using trigonometrical equations

For San Diego, California, a simple equation to model daylight hours would be:

Number of daylight hours $= 2.4\sin(0.017t - 1.377) + 12$

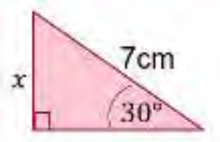
where t is the day of year from 0 to 365



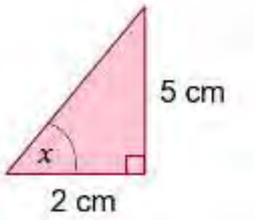
- From the graph can you tell which dates of the year are the shortest and longest day?

Solving Equations with Trigonometry

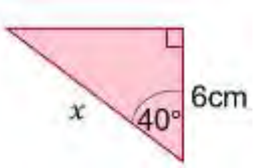
1. Calculate the length of the side marked x in this triangle.



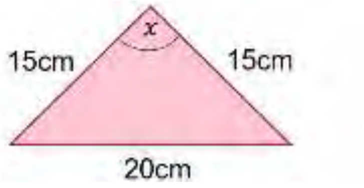
2. Calculate the value of the angle marked x in this triangle.



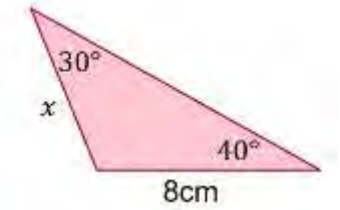
3. Calculate the value of the side marked x in this triangle



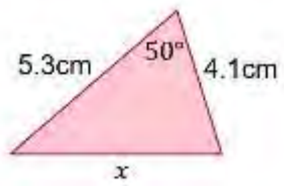
4. Calculate the value of the angle marked x in this triangle.



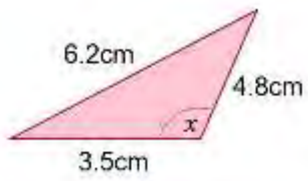
5. Calculate the value of the side marked x in this triangle



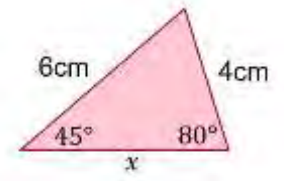
6. Calculate the value of the side marked x in this triangle.



7. Calculate the value of the angle marked x in this triangle.



8. Calculate the value of the side marked x in this triangle.

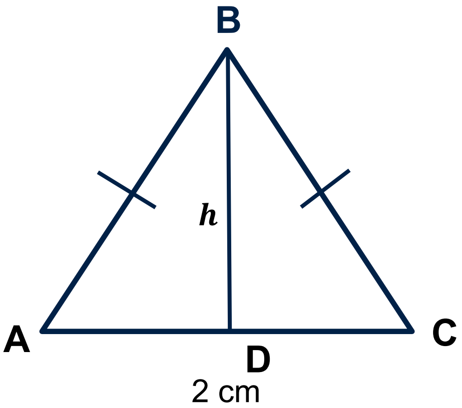


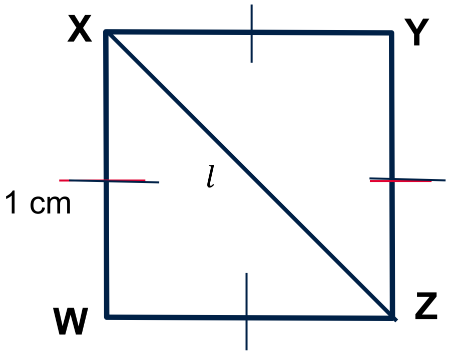
Other Equations

Solve the following

1. $3^x = 243$
2. $2^{2x+3} = 128$ **Hint:** write 128 as powers of 2
3. $\sqrt{x+3} = 7$
4. $2\sqrt{x} + 1 = \sqrt{12} + 3$
5. $3\sqrt{x} + 12 = 7\sqrt{x}$
6. $\sin x = \frac{1}{2} \quad 0 \leq x \leq 360$
7. $\cos x = 0.866 \quad 0 \leq x \leq 360$
8. $\frac{8}{3x+7} = 2$

Missing info





	Answer
Length of AB	
Length of BD	
Length of AD	
Size of $\angle BAD$	
Size of $\angle ABD$	

	Answer
Length of WZ	
Length of XZ	
Size of $\angle WZX$	
Size of $\angle WXZ$	

Use your knowledge of regular shapes to complete the tables above (you will need them for the next task).

Let's get Triggy

Use your tables and diagrams from the previous activity to complete this table

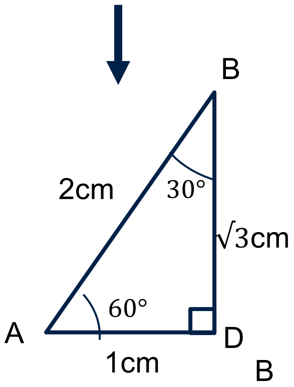
θ	$30°$	$45°$	$60°$
$\sin\theta$	$\frac{}{AB}=\frac{1}{2}$	$\frac{XW}{}=\frac{WZ}{XZ}=\frac{}{}$	$\frac{}{AB}=\frac{}{}$
$\cos\theta$	$\frac{}{}=\frac{\sqrt{3}}{}$	$\frac{}{}=\frac{WZ}{}=\frac{}{}$	$\frac{}{}=\frac{}{}$
$\tan\theta$	$\frac{}{}=\frac{}{\sqrt{3}}$	$\frac{}{}=\frac{}{}=1$	$\frac{}{}=\frac{}{1}=\sqrt{}$

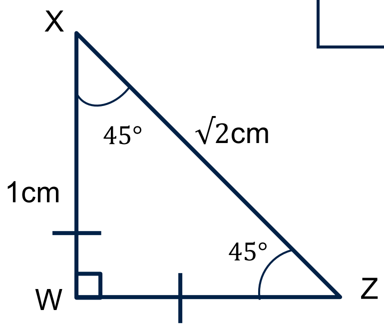
Let’s get Triggy Hint

Use your tables and diagrams from the previous activity to complete this table

Some examples are filled in to get you started

These will help

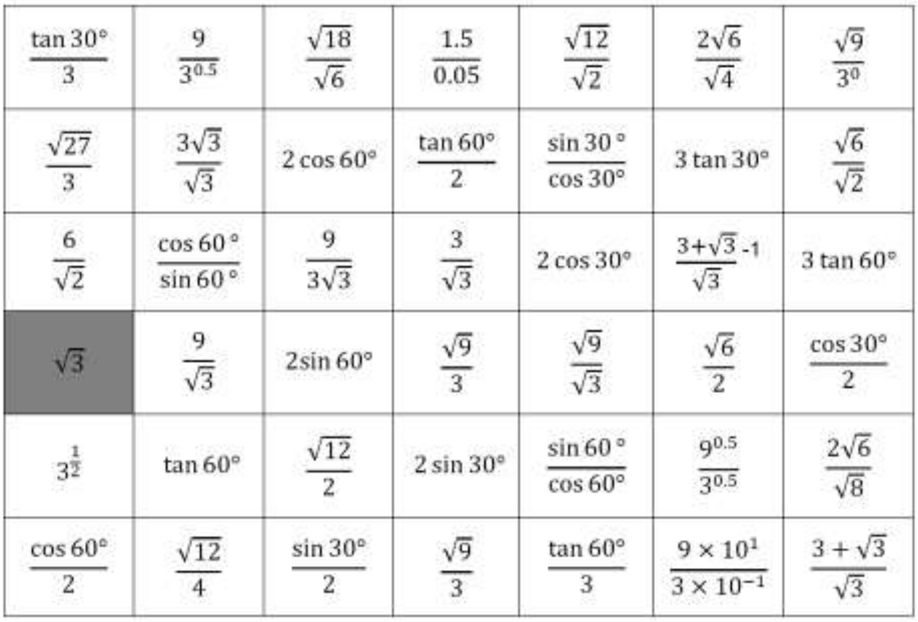




θ	30°	45°	60°
$sin\theta$ =	$\frac{AD}{AB}=\frac{1}{2}$	$\frac{XW}{\quad}=\frac{WZ}{XZ}=\frac{\quad}{\quad}$	$\frac{\quad}{AB}=\frac{\sqrt{3}}{\quad}$
$cos\theta$ =	$\frac{\quad}{\quad}=\frac{\sqrt{3}}{\quad}$	$\frac{\quad}{\quad}=\frac{WZ}{XW}=\frac{\quad}{\quad}$	$\frac{AD}{AB}=\frac{\quad}{\quad}$
$tan\theta$ =	$\frac{\quad}{\quad}=\frac{\quad}{\sqrt{3}}$	$\frac{\quad}{\quad}=\frac{\quad}{\quad}=1$	$\frac{\quad}{\quad}=\frac{\quad}{1}=\sqrt{\quad}$

Trig Maze

Starting at $\sqrt{3}$ on the left hand side of the rectangle, find your way to the right hand side by landing only on expressions that are equivalent to $\sqrt{3}$



$\frac{\tan 30°}{3}$	$\frac{9}{3^{0.5}}$	$\frac{\sqrt{18}}{\sqrt{6}}$	$\frac{1.5}{0.05}$	$\frac{\sqrt{12}}{\sqrt{2}}$	$\frac{2\sqrt{6}}{\sqrt{4}}$	$\frac{\sqrt{9}}{3^0}$
$\frac{\sqrt{27}}{3}$	$\frac{3\sqrt{3}}{\sqrt{3}}$	$2\cos 60°$	$\frac{\tan 60°}{2}$	$\frac{\sin 30°}{\cos 30°}$	$3\tan 30°$	$\frac{\sqrt{6}}{\sqrt{2}}$
$\frac{6}{\sqrt{2}}$	$\frac{\cos 60°}{\sin 60°}$	$\frac{9}{3\sqrt{3}}$	$\frac{3}{\sqrt{3}}$	$2\cos 30°$	$\frac{3+\sqrt{3}-1}{\sqrt{3}}$	$3\tan 60°$
$\sqrt{3}$	$\frac{9}{\sqrt{3}}$	$2\sin 60°$	$\frac{\sqrt{9}}{3}$	$\frac{\sqrt{9}}{\sqrt{3}}$	$\frac{\sqrt{6}}{2}$	$\frac{\cos 30°}{2}$
$3^{\frac{1}{2}}$	$\tan 60°$	$\frac{\sqrt{12}}{2}$	$2\sin 30°$	$\frac{\sin 60°}{\cos 60°}$	$\frac{9^{0.5}}{3^{0.5}}$	$\frac{2\sqrt{6}}{\sqrt{8}}$
$\frac{\cos 60°}{2}$	$\frac{\sqrt{12}}{4}$	$\frac{\sin 30°}{2}$	$\frac{\sqrt{9}}{3}$	$\frac{\tan 60°}{3}$	$\frac{9\times 10^1}{3\times 10^{-1}}$	$\frac{3+\sqrt{3}}{\sqrt{3}}$

Triggy Problems

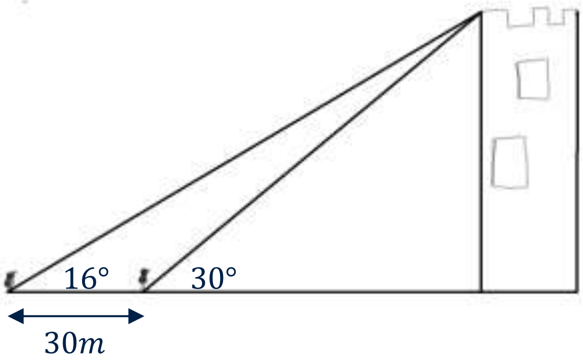
1. The area of an equilateral triangle is $10\ cm^2$.

   What are the lengths of the sides?

2. Two birds are sitting looking at the top of a tower block, as shown in the diagram

   They are 30m apart.

   How tall is the tower?



Multiple Equations

$$\text{If } \frac{ab}{a+b} = \frac{1}{4} \text{ and } \frac{bc}{b+c} = \frac{1}{2} \text{ and } \frac{ac}{a+c} = \frac{1}{8} \quad \text{find } a,\ b \text{ and } c$$

Hint:

- Rearrange these equations so they are linear i.e. no fractions
- Find an expression for b and c in terms of a
- Substitute into the equation that uses b and c

Powers

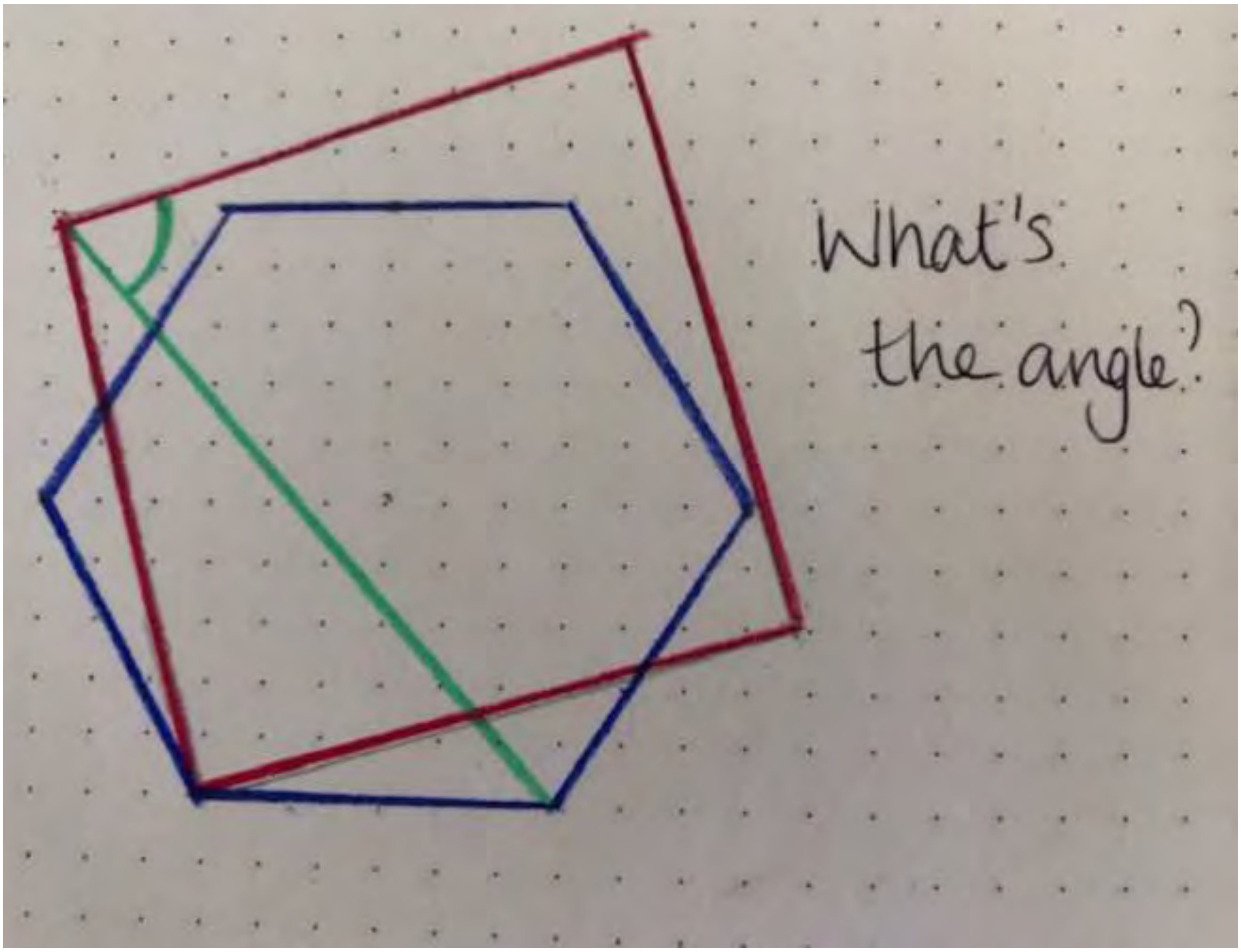
Using what you know about powers, can you solve this equation

$$(x-6)^{x^2-9}=1$$

Hint

- What do you know about a^0
- What do you know about 1^a
- What do you know about $(-1)^a$

Geometry Puzzle



Solving Solutions

Linear Equations

Solving Linear 1 Solutions

1. $x = 16/3$
2. $3 < x < 4$
3. $x > -2$
4. $x = 5$
5. $6/5 \geq x \ or \ x \leq 6/5$
6. $-4 < x \ or x > -4$
7. $x = 2$
8. $x = 10/3$

Solving Linear 2 Solutions

1. $x = 7$
2. $x = 4.5$
3. $x = 6$
4. $x > 7/4$
5. $-3/2 < x < -1$
6. $x = 4$
7. $x \leq -3$
8. $x = 25$

Piggy in the Middle

5	11	17	23	29

Chicken Run

Width (m)	**Length (m)**	**Area (m^2)**
6	9	54
5	8	40
4	7	28

So, Victoria can make 3 different sized enclosures with an area greater than 20m^2

Solving Solutions

Crack the Code

Solving the equations in the following order:

$g = 15$	$c = 9$	$a = 6$
$y = 5$	$r = 2$	$m = 7$
$s = 3$	$k = 8$	$h = 1$
$d = 10$	$10 < t < 13$	$e < 4.16 \ldots$

Reveals: "The Mystery Message Cracked"

Simultaneous Equations

1 $x = 2,\ y = 3$

2 $x = 3,\ y = -1$

3 $x = 2,\ y = 11$

4 $x = \frac{1}{2},\ y = -2$

Maths at the Movies

Number of rentals	1 575 000
Number of sales	370 000

Taxi

Distance travelled	12 miles
Value of x	£2

Solving Graphically

1. $x = 2,\ y = 4$ 2. $x = 4,\ y = -2$ 3. $x = 8,\ y = 2$

Puzzle to Ponder

If you rearrange the second equation you get $y = 2x + 8$

Both equations have the same gradient so they are parallel, they will never meet

Solving Solutions

Triple Simultaneous Equations

$x = 3,\ \ y = 1,\ \ z = 6$

Mean Problem

Adding you get $5x + 5y + 5z = 1155$

Dividing by 5 $\qquad x + y + z = 231$

Dividing by 3 (3 terms) you get the mean which is 77

Quadratic Equations

Solving Quadratics 1

1. $x = \pm 4$
2. $x = 0 \text{ or } x = 16$
3. $x = -1 \text{ or } x = \frac{3}{2}$
4. $x = 2 \text{ or } x = 1$
5. $x = \frac{5}{2} \text{ or } x = -\frac{3}{4}$
6. $x = \frac{1}{3} \text{ or } x = -5$
7. $x = 2 \text{ or } x = -8$
8. $x = \frac{3}{2} \text{ or } x = \frac{1}{5}$

Solving Quadratics 2

1 $\quad x = 6 \text{ or } x = -2$

2 $\quad x = 3 \text{ or } x = -2$

3 $\quad x = \frac{3}{2} \text{ or } x = 4$

4 $\quad x = -\frac{3}{2} \text{ or } x = \frac{4}{3}$

5 $\quad x = 3 \text{ or } x = -1$

6 $\quad x = 2 \pm \sqrt{5}$

7 $\quad x = -\frac{1}{3} \text{ or } x = \frac{2}{3}$

8 $\quad x = 2\ \ (Note\ x \neq -\frac{17}{6})$

Quadthagoras

The only solution in context is $x = 5$

Suitable lengths are width = 5, length = 12, diagonal = 13

Up In the Air

The object strikes the ground after 6 seconds (only consider positive answer as the time after the launch

Solving Solutions

Using Graphs

1. $4 + 3x - x^2 = 0 \qquad x = -1 \; or \; x = 4$

2. $x^2 - 6x + 8 = 0 \qquad x = 2 \; or \; x = 4$

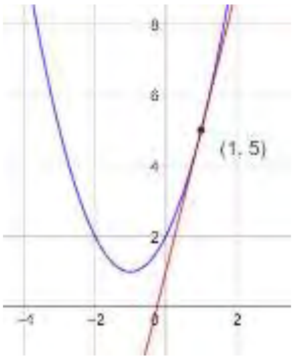
3. $3x^2 - 3x - 6 = 0 \qquad x = -1 \; or \; x = 2$

4. $4 + 3x - x^2 = 4 \qquad x = 0 \; or \; x = 3$

Simultaneously

1 $\quad x = 2 , y = 7 \quad or \;\; x = -5, y = -14$

2 $\quad x = 1, y = 5$



Problem 1: When $a = \frac{10}{3}, b = \frac{8}{3}$ or when $a = 2, b = 8$

Problem 2: $(-\frac{17}{5}, -\frac{24}{5})$ and $(1,4)$

Lines and Curves

$A\,(1, -1) \qquad B\,(4\,, 2) \qquad AB = \sqrt{18} \;\; or \;\; 3\sqrt{2} \qquad BC = \sqrt{8} \;\; or \;\; 2\sqrt{2}$

Area of triangle ABC is 6 square units

Solving Other Equations

Solving equations with trigonometry

1. $x = 3.5cm$

2. $x = 68.2° \; to \; 1 \; d.p$

3. $x = 7.8cm \; to \; 1 \; d.p$

4. $x = 83.6° \; to \; 1 \; d.p$

5. $x = 10.3 \; cm \; to \; 1 \; d.p$

6. $x = 4.1 \; cm \; to \; 1 \; d.p$

7. $x = 95.4° \; to \; 1 \; d.p$

8. $x = 5.0cm \; to \; 1 \; d.p$

Solving Solutions

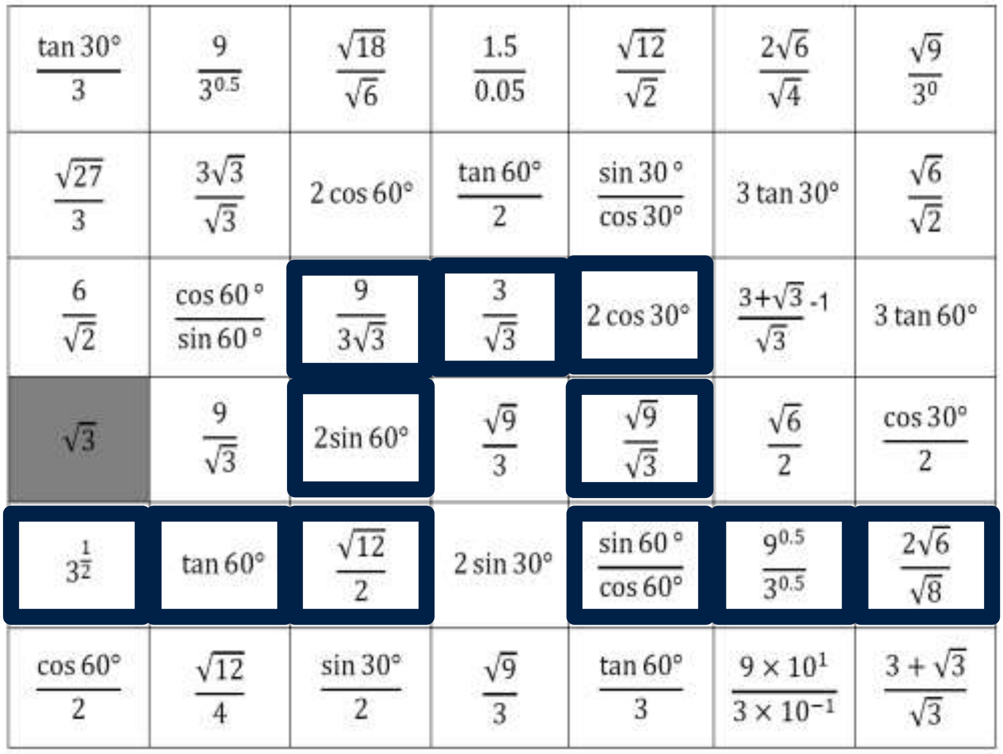
Solving Other Equations

1. $x = 5$
2. $x = 2$
3. $x = 46$
4. $x = 4 + 2\sqrt{3}$
5. $x = 9$
6. $x = 30° \text{ or } x = 150°$
7. $x = 30° \text{ or } x = 330°$
8. $x = -1$

Let's get Triggy

θ	30°	45°	60°
$sin\theta$	$\frac{AD}{AB} = \frac{1}{2}$	$\frac{XW}{XZ} = \frac{WZ}{XZ} = \frac{1}{\sqrt{2}}$	$\frac{BD}{AB} = \frac{\sqrt{3}}{2}$
$cos\theta$	$\frac{BD}{AB} = \frac{\sqrt{3}}{2}$	$\frac{WX}{WZ} = \frac{WZ}{WX} = \frac{1}{\sqrt{2}}$	$\frac{AD}{AB} = \frac{1}{2}$
$tan\theta$	$\frac{AD}{BD} = \frac{1}{\sqrt{3}}$	$\frac{WX}{WZ} = \frac{WZ}{WX} = 1$	$\frac{BD}{AD} = \frac{\sqrt{3}}{1} = \sqrt{3}$

Trig Maze



$\frac{\tan 30°}{3}$	$\frac{9}{3^{0.5}}$	$\frac{\sqrt{18}}{\sqrt{6}}$	$\frac{1.5}{0.05}$	$\frac{\sqrt{12}}{\sqrt{2}}$	$\frac{2\sqrt{6}}{\sqrt{4}}$	$\frac{\sqrt{9}}{3^0}$
$\frac{\sqrt{27}}{3}$	$\frac{3\sqrt{3}}{\sqrt{3}}$	$2\cos 60°$	$\frac{\tan 60°}{2}$	$\frac{\sin 30°}{\cos 30°}$	$3\tan 30°$	$\frac{\sqrt{6}}{\sqrt{2}}$
$\frac{6}{\sqrt{2}}$	$\frac{\cos 60°}{\sin 60°}$	$\frac{9}{3\sqrt{3}}$	$\frac{3}{\sqrt{3}}$	$2\cos 30°$	$\frac{3+\sqrt{3}}{\sqrt{3}}-1$	$3\tan 60°$
$\sqrt{3}$	$\frac{9}{\sqrt{3}}$	$2\sin 60°$	$\frac{\sqrt{9}}{3}$	$\frac{\sqrt{9}}{\sqrt{3}}$	$\frac{\sqrt{6}}{2}$	$\frac{\cos 30°}{2}$
$3^{\frac{1}{2}}$	$\tan 60°$	$\frac{\sqrt{12}}{2}$	$2\sin 30°$	$\frac{\sin 60°}{\cos 60°}$	$\frac{9^{0.5}}{3^{0.5}}$	$\frac{2\sqrt{6}}{\sqrt{8}}$
$\frac{\cos 60°}{2}$	$\frac{\sqrt{12}}{4}$	$\frac{\sin 30°}{2}$	$\frac{\sqrt{9}}{3}$	$\frac{\tan 60°}{3}$	$\frac{9 \times 10^1}{3 \times 10^{-1}}$	$\frac{3+\sqrt{3}}{\sqrt{3}}$

Solving Solutions

Triggy Problems

Problem 1	Problem 2
$x = 4.806 \; to \; 3 \; s.f$	$17.1m \; to \; 3 \; s.f$

Multiple Equations

$a = \frac{1}{5}, \qquad b = -1, \qquad c = \frac{1}{3}$

Powers

Case 1: Power is zero so $x = \pm 3$

Case 2 base is 1 so $x = 7$

Case 3 base is -1 so $x = 5$

Geometry Puzzle

Angle is $60°$

Linear Sketching

Did you know?

Where is the steepest street in the world?

Gradients can be represented in different ways, but what do the measurements mean?

A gradient of $1:5$ means for every 5m you travel horizontally you travel 1m vertically.

A gradient of 16% means for every 100m across you go 16m up.

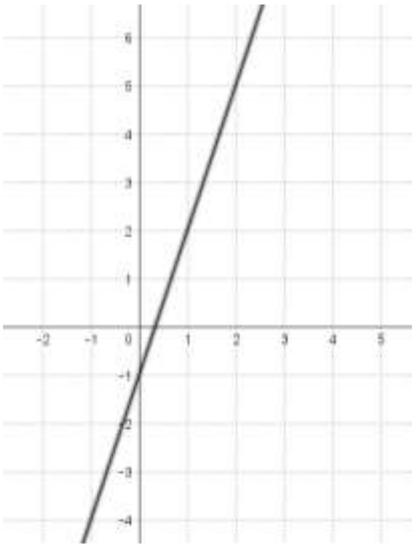
Can you find out:

- Where the street is?
- What the gradient of the street is?
- Why there is controversy over the winners?

Clue: They hold a Jaffa rolling contest down the street every year!

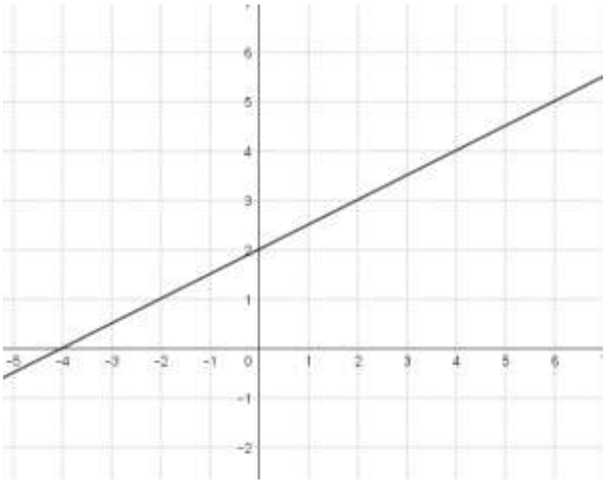
Linear Graphs 1

1. What are the gradient and intercept of the line $y = 3x - 5$?
2. Find the gradient of the line connecting (3,10) and (1,6)
3. Find the midpoint between the points (3,-8) and (-1,4)
4. Find the distance between points (1,10) and (4,18)
5. What is the equation of the line with gradient 3 that passes through (5,8)?
6. Does the line $y = 2x - 3$ pass through (1,-1)? Explain how you know.
7. Find the equation of a line that is parallel to $y = 5x - 2$ that passes through (2,19)
8. What is the equation of this graph?



Linear Graphs 2

1. What are the gradient and y intercept of the line $y = 2x - 7$?
2. Find the gradient of the line connecting (1,4) and (-1,0)
3. Find the midpoint between the points (-2,10) and (6,4)
4. Find the distance between the points (4,11) and (-1,15)
5. What is the equation of the line with gradient 2 that passes through (1,4)?
6. Does the line $y = -2x + 5$ pass through (3,1)? Explain how you know.
7. Find the equation of a line that is parallel to $y = -\frac{3}{2}x - 1$ that passes through (6,4)
8. What's the equation of this graph?



Do they cross?

Line A passes through the points $(-3,1)$ and $(3,5)$
Line B passes through the points $(0,-4)$ and $(6,4)$

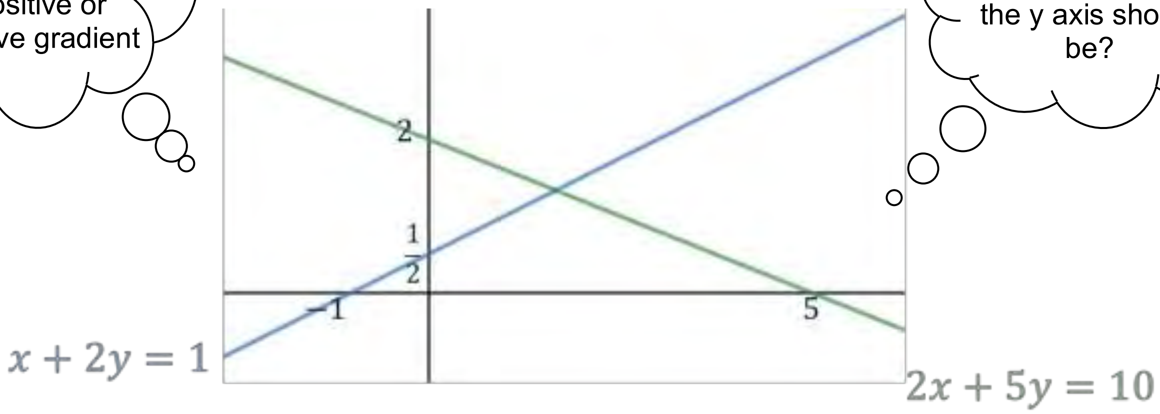
- By sketching can you tell if the lines will meet?
- If they do meet what are the points of intersection?
- Challenge! Can you find where the lines will meet using algebra

Picture this

Think about whether the equation suggests a positive or negative gradient

Is this an accurate sketch of these two lines?

What does the equation suggest the intercept with the y axis should be?



The plot thickens….

Complete the information in the table for each equation below:

- Find the co-ordinates of the x and y intercepts
- Decide if the gradient of the graph would be positive or negative

Name	Equation	x intercept	y intercept	Positive/negative gradient
A	$y - 2x - 1 = 0$			
B	$y = 3$			
C	$3x + 4y = 2$			
D	$2x - y + 6 = 0$			
E	$2y + x = 4$			
F	$2x + y - 3 = 0$			

Using the information from the table, sketch all the graphs on one set of axes to find:

- A pair of lines that are parallel
- A pair of lines that are perpendicular
- A pair of lines that intersect at $(-2,\ 2)$

Two geometry problems

DEF is an isosceles right angled triangle
The line passing through D and F has the equation
$x + 3y = 15$
D is the co-ordinate $(6{,}3)$
E is the co-ordinate $(5{,}0)$
The angle EDF is the right angle

Can you find:

- The equation of line DE?
- The possible coordinates of F?
- The equation of line EF?

ABCD is a parallelogram. The line passing through C and D has the equation $y = 7$
The line CD is 5 units long
D has coordinate $(2{,}7)$
C has both positive x and y co-ordinates
The line through AC has equation $3x + 2y = 35$
A has coordinate $(9{,}4)$
Can you find:

- The coordinate of C?
- The equation of line AB?
- The equation of line BD?
- The area of the parallelogram?

Sketching Linear Inequalities

Sketch and shade the following inequalities.

1. $y \leq 6$ 2. $x < 6$ 3. $x + 2y \geq 8$ 4. $3x + 2y \geq 12$

- Shade out the side of the line that doesn't satisfy the inequality.
- Label the correct region **R**

Geometry from equations

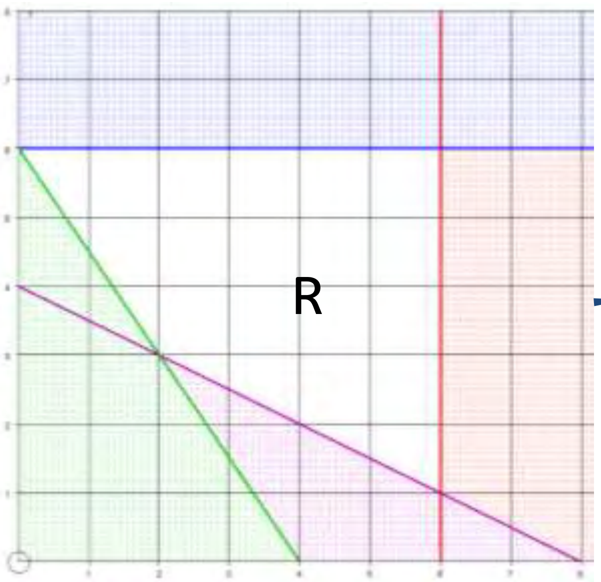
The following equations enclose a square:

$$
\begin{gathered}
y - 2 = x \\
y + x = 6 \\
y = x - 1 \\
y + x - 3 = 0
\end{gathered}
$$

- Which are the two pairs of parallel sides?
- What are the coordinates of all 4 vertices
- How can you convince yourself this is a square?

Linear Programming

Here is a graph that shows the feasible region R satisfied by the all inequalities from the previous question.



In Linear Programming linear inequalities are used to find solutions to real life problems.

The ‘optimal’ or best solution for is found for a particular objective.

R, the unshaded region is called the FEASIBLE REGION. Points in this region satisfy all of the inequalities.

The feasible region has four vertices What are the coordinates?

- Use the diagram to have a go at this question

Maximise the value of $x + y$ within the region satisfied by the inequalities:

$$x + 2y \geq 8, \qquad 3x + 2y \geq 12, \qquad y \leq 6, \qquad x \leq 6$$

Catching Stars

Go to Student.Desmos.com (use classroom code: **3VJUM2**) to try a Linear Marbleslides Challenge.

You will be investigating the features of linear graphs whilst trying to catch as many stars as possible.

You can join the activity without signing in or entering your real name.

Sketching Quadratic Graphs

Did you know?

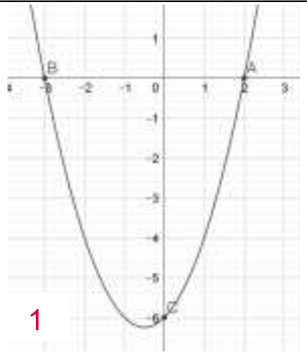
Quadratic curves are also known as parabolas.

Parabolas are used in many examples of architecture.

- Do you recognise these landmarks?
- Do you think they are parabolas?
- Can you find any more examples of architecture that use parabolas near where you live?

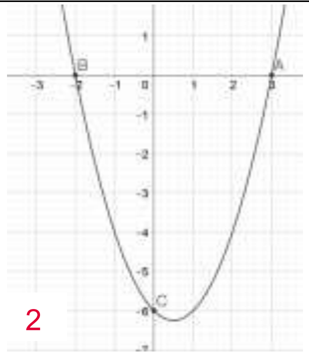
Quadratic Graphs 1

Find the coordinates of A, B and C on each graph



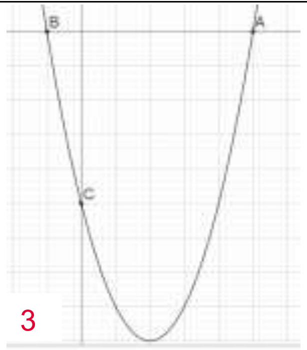
1

$y=(x+3)(x-2)$



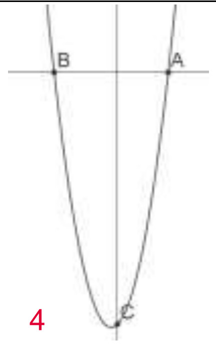
2

$y=(x-3)(x+2)$



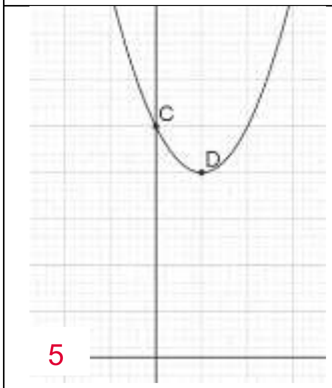
3

$y=(x-5)(x+1)$



4

$y=(x+5)(x-4)$

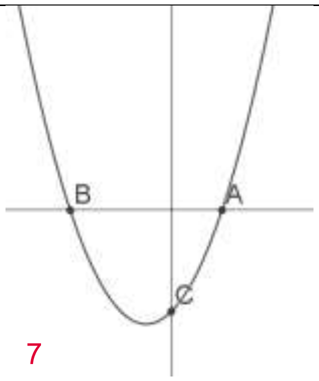


5

$y=(x-1)^2+4$

6

$y=(x+3)^2+7$



7

$y=x^2+3x-4$

8

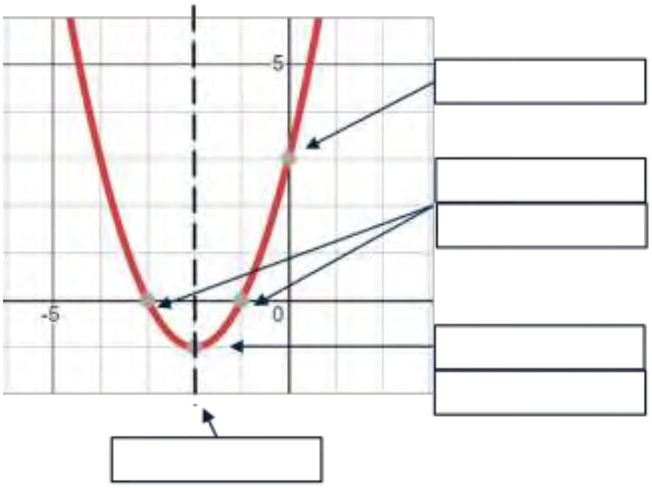
$y=2x^2+4x-6$

What is a sketch?

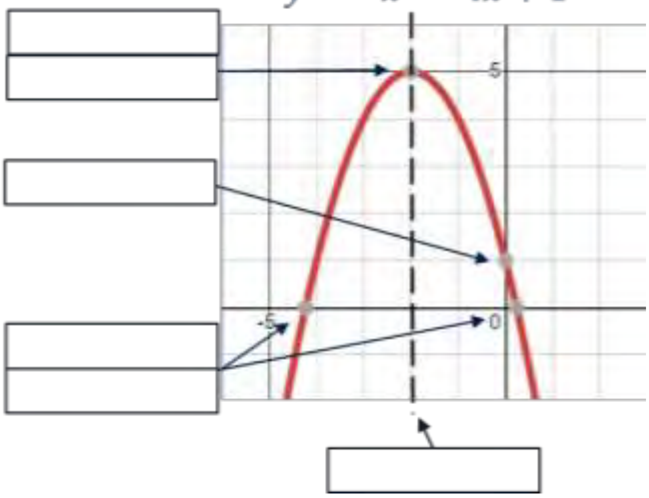
In mathematics a sketch does not need to be a completely accurate drawing, but it should "illustrate all the significant features of the graph/shape"

These diagrams show the key features of a quadratic graph

$y=x^2+4x+3$



$y=-x^2-4x+1$



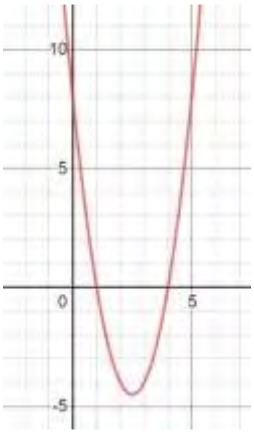
Put the words below into the boxes above so that the quadratic graphs are labelled correctly. Some words may be used more than once.

x intercepts
minimum
roots
turning point
maximum
axis of symmetry
y intercept

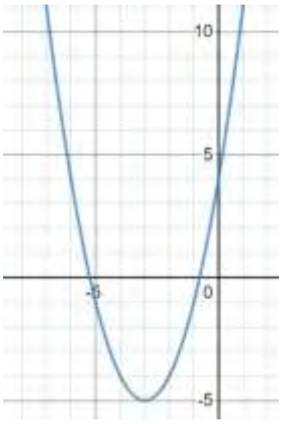
Identification Parade

Which of the graphs below is $y = x^2 - 5x + 4$?

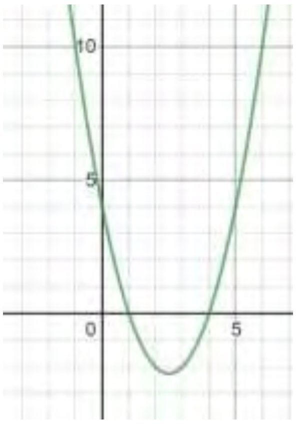
A



B



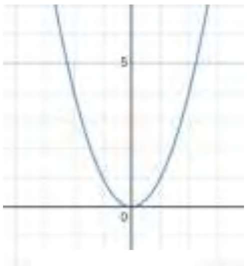
C



Move it!

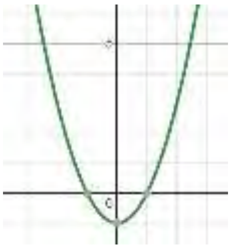
- Can you describe how to move Graph A onto Graph B?

GRAPH A



$y = x^2$

GRAPH B



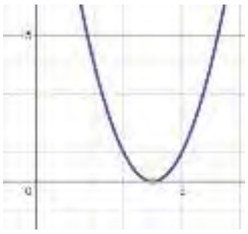
$y = x^2 - 1$

- Which transformations would take GRAPH A onto each of the graphs below?

GRAPH C

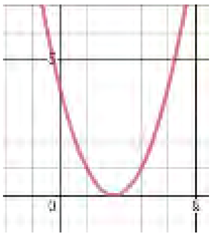
$y = x^2 + 3$

GRAPH D



$y = (x - 4)^2$

GRAPH E

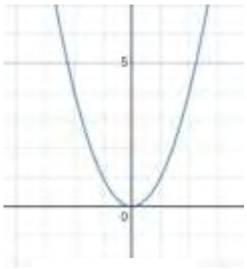


$y = (x - 2)^2$

Move it again!

- Can you describe how to move Graph A onto Graph F?

GRAPH A



$y = x^2$

GRAPH F

$y = (x-4)^2 + 3$

- Can you see how that links to the equation of the graph?

Complete the square to get sorted!

Below are ten quadratic equations.

$y = x^2 - 4x + 7$

$y = x^2 + 2x + 5$

$y = x^2 - 6x + 16$

$y = x^2 - 6x + 25$

$y = x^2 - 2x + 5$

$y = x^2 + 8x + 20$

$y = x^2 - 6x + 11$

$y = x^2 - 8x + 21$

$y = x^2 + 6x + 10$

$y = x^2 - 10x + 29$

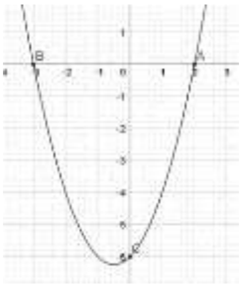
Your task is to place **nine** of them into a 3 by 3 grid according to the rules on these cards

All of the equations in the top row have a turning point on the line $y = 4$	One of the equations in the left hand column has its turning point at (-4, 4)	The equation with a turning point at (-1, 4) is not on the top row	The equations in the top left and centre right square both have the same y coordinate for their turning point	All of the coordinates of the turning points are at integer values of x and y. None of the turning points are on either axis.
All three of the equations with a turning point on the line $x = 3$ are on the bottom row	All of the equations in the centre column have turning points on the line $y = 5 - x$	All of the turning points for the equations in the centre column are in the first quadrant	$y = x^2 + 6x + 10$ is in the square in the left hand column directly above $y = x^2 - 6x + 16$	The turning point of the equation $y = (x + a)^2 + b$ is at $(-a, b)$

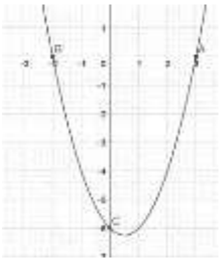
Quadratic Graphs 2

1. What are the x intercepts of y=(2x+3)(x+4)?

2. What are the x and y intercepts of this graph.



3. Write the equation of the graph in the form $ax^2 + bx + c$



4. What are the x intercepts of the graph of $y = 6x^2 + x \ - 2$?

5. What does the c part of the equation in $y = ax^2 + bx + c$ represent on a graph?

6. Sketch the graph of $y = 3x^2 - 2x - 8$
Label the x and y intercepts

7. What are the coordinates of the points marked on the diagram of the equation $y = x^2 + 6x + 16$?

8. Which of these statements about the graph $y = x^2 - 4x + 8$ are true

Has a minimum point at (2, 4)

Will not cross the x axis twice

Can be factorised

How High?

The height of a ball thrown up from the ground into the air at time, t, is given by:

$$\boldsymbol{h = 20t - 10t^2}$$

- Find when the ball hits the ground
- How long is the ball more than 5m above the ground?
- Find the maximum height reached by the ball

Don't forget a sketch will help!

Inequalities reminder

Solve the inequality $x^2 - 4x - 5 \geq 0$

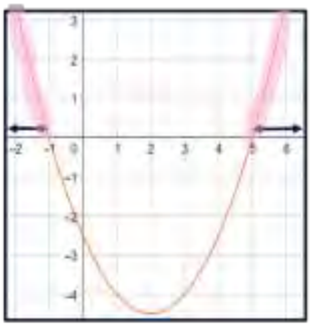
Rearrange into factorised form…

$(x-5)(x+1) \geq 0$

…now you can sketch the graph

When $x < -1$ and $x > 5$ the curve is above the x axis.

This is where $x^2 - 4x + 5 \geq 0$



These are two regions, so are represented by two inequalities

$x < -1$ and $x > 5$

Solve the inequality $x^2 - 4x - 5 < 0$

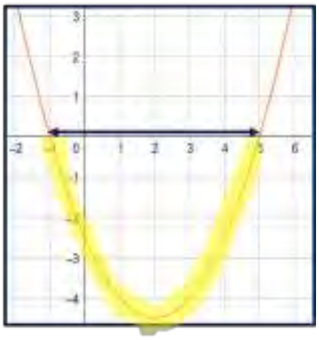
Rearrange into factorised form…

$(x-5)(x+1) < 0$

…now you can sketch the graph

When $x > -1$ and $x < 5$ the curve is below the x axis.

This is where $x^2 - 4x + 5 < 0$



This is one region, so can be represented by one inequality

$-1 < x < 5$

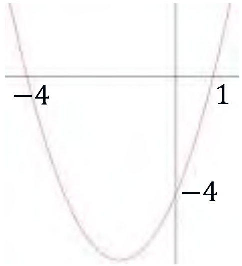
Quadratic Inequalities

- Use a sketch to help you solve the following inequalities

1. $(x-2)(x+3) < 0$
2. $(4+x)(2-x) < 0$
3. $x^2 + 7x + 12 \geq 0$
4. $36 \geq (x+2)^2$

Fill the table

- Complete the rows in the table with the information that you have been given

Sketch	Equation	x intercept	y intercept	Minimum point
				
		$(5,0)$ $(-2,0)$	$(0,-10)$	
				(-5, 6)
	$y=x^2+6x+8$		$(0,8)$	

Catching Stars

Go to Student.Desmos.com (use classroom code: **E96QV4**) to try a Quadratic Marbleslides Challenge.

You will be investigating the features of quadratic graphs whilst trying to catch as many stars as possible.

You can join the activity without signing in or entering your real name.

Sketching Other Graphs

Did you know?

Trigonometric functions can be used to model many things that repeat over a time period.

Examples include: Tides, springs, harmonic strings and hours of daylight.

Sketching Other Graphs 1

1. What is the mathematical name for the graph of $y = \frac{1}{x}$?
2. What are the maximum and minimum values for the graph $y = cos\theta$?
3. Sketch the graph of $y = 2^x$. Label the y and x intercepts.
4. Using a sketch of the graphs

   $$y = \frac{1}{x} \text{ and } y = x$$

   Show how many solutions there will be to the equation $\frac{1}{x} = x$
5. What is the name for this type of graph?
6. What is the y intercept of the graph $y = (x + 2)(x - 3)(x + 5)$?
7. What are the x intercepts of the graph $y = (x + 2)(x - 3)(x + 5)$?
8. Sketch the graph of $y = (x - 3)(x + 2)(x + 5)$

Sketching Other Graphs 2

1. What is the mathematical name for graphs of the form $x^2 + y^2 = 9$?
2. Sketch the graph of $y = sin\theta$ between $0°$ and $360°$, labelling x and y intercepts
3. On your sketch for Q2 draw in the line

   $$y = 0.5$$

   How many solutions are there to

   $$sin\theta = 0.5?$$

   Can you say what they are?
4. Sketch the graph $y = x^3$, labelling any intersections
5. Sketch the graph of the equation in Q1, label any intersections with the x and y axis
6. What is the y intercept of the graph $y = (x + 1)(x + 1)(x - 1)$?
7. What are the x intercepts of the graph $y = (x + 1)(x + 1)(x - 1)$?
8. Sketch the graphs of

   $$x^2 + y^2 = 4$$

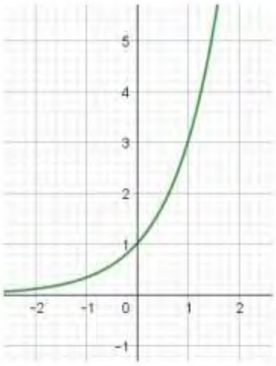
   $$y = x + 1$$

   Use the sketch to determine how many solutions there are when those equations are solved simultaneously

Which is which?

Match the graphs to the equations - there are more equations than you need!

A



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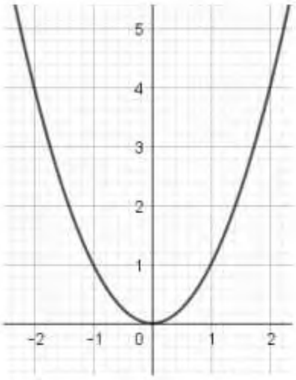
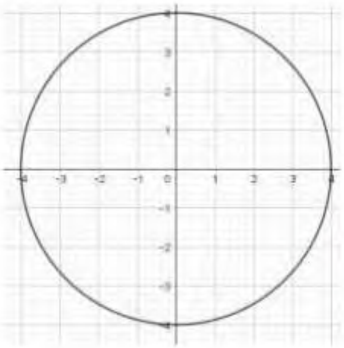
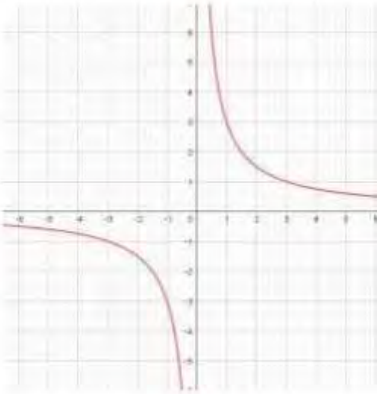


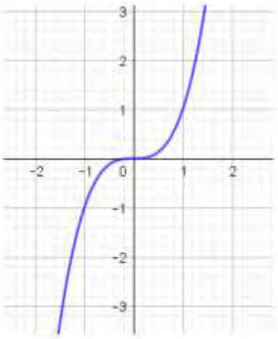
C



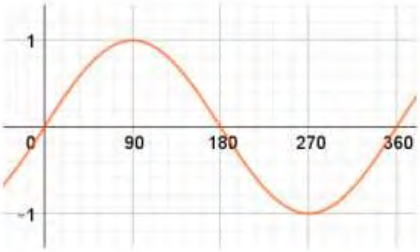
D



E



F



- $y = 3^x$
- $y = \frac{3}{x}$
- $y = sin\theta$
- $y = 2x^2$
- $y = tan\theta$
- $y^2 + x^2 = 16$
- $y = x^2$
- $y = x^3$
- $y = \frac{1}{x}$
- $y = x^2 + 3$

Shortest Distance

Find the shortest distance between the following curves:

$$x^2 + y^2 = 9$$
$$y = x^2 + 7$$

How fast?

A car is initially travelling at 300m/min, it speeds up over a 20 second interval with a constant acceleration to achieve a speed of 400m/min.

It travels at this speed for 3 minutes before slowing to a stop via constant de-acceleration over a period of 30 seconds.

- What is the car's average speed for the first 20 seconds of travel?
- What is the car's deceleration?

A square in a circle.

A square is placed inside a circle (C_1) so that the corners perfectly touch the circle's circumference.

Another circle (C_2) is now placed inside this square so that its circumference perfectly touches the square's sides.

What is the ratio of the lengths of the radius of C_1 and the radius of C_2 ?

Hint: Assume C_2 has a radius of 1 unit

A Triggy Problem!

Solve $(\sin x + 1)(2\cos x - 1) = 0$ for $0 < x < 360°$

A cubic match up

Which one of the equations below describes the graph?

1. $y = (x + 1)(x - 1)(x - 2)$
2. $y = -x(x - 1)(x + 1)$
3. $y = x(x - 1)\,(x + 1)$

Catching Stars

Go to Student.Desmos.com (use classroom code: **FENFZP**) to try an Exponential Marbleslides Challenge.

You will be investigating the features of exponential graphs whilst trying to catch as many stars as possible.

You can join the activity without signing in or entering your real name.

Sketching Solutions

Linear Sketching

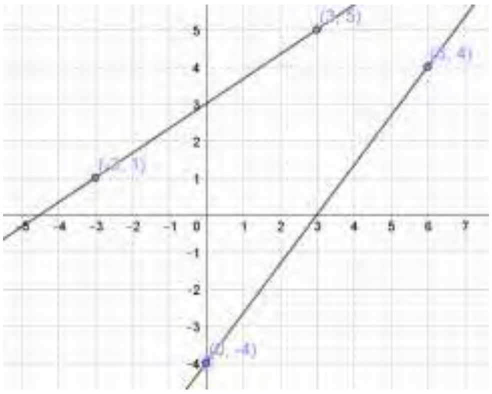
Linear Graphs 1

1. Gradient = 3, intercept = -5
2. Gradient = $\frac{10-6}{3-1} = 2$
3. Midpoint = $(\frac{3+-1}{2}, \frac{-8+4}{2}) = (1, \ -2)$
4. Distance = $\sqrt{(4-1)^2 + (18-10)^2} \ = \sqrt{73}$
5. Equation is $y = 3x - 7$
6. $y = 2 \times 1 \ - 3 = -1$
    a. Yes the line passes through (1,-1)
7. Equation is y=5x+9
8. $y = 3x - 1$

Linear Graphs 2

1. Gradient = 2, intercept = -7
2. Gradient = $\frac{4-0}{1--1} = 2$
3. Midpoint = $(\frac{-2+6}{2}, \frac{10+4}{2}) = (2,7)$
4. Distance = $\sqrt{(4--1)^2 + (11-15)^2} = \sqrt{41}$
5. Equation is $y = 2x + 2$
6. No, the line doesn't pass through (3,1)
   as when $x = 3, y = -1$
7. Equation is $y = -\frac{3}{2}x + 13$
8. $y = \frac{1}{2}x + 2$

Do they cross?



From a sketch we can see that the lines are not parallel.

They will meet at some point

The equations of the lines are $y = \frac{4}{3}x - 4$ and $y = \frac{2}{3}x + 3$

Solving simultaneously we find that the lines intersect at (10.5, 10)

Sketching Solutions

Picture this

$x + 2y = 1$ should have a negative gradient, which it doesn't in the sketch
Also, the y intercept is $\left(0, \frac{1}{2}\right)$, the x intercept is $\left(\frac{1}{1}, 0\right) = (1,0)$
So they have sketched $-x + 2y = 1$

$2x + 5y = 10$ should have a negative gradient, which it does.
The y intercept is $\left(0, \frac{10}{5}\right) = (0,2)$ and the x intercept is $\left(\frac{10}{2}, 0\right) = (5,0)$
So this line is correct

The plot thickens….

Name	Equation	x intercept	y intercept	Positive/negative gradient
A	$y - 2x - 1 = 0$	$(-\frac{1}{2}, 0)$	$(0,\ 1)$	Positive
B	$y = 3$	No intercept	$(0,\ 3)$	Horizontal line
C	$3x + 4y = 2$	$(\frac{2}{3},\ 0)$	$(0, \frac{1}{2})$	Negative
D	$2x - y + 6 = 0$	$(-3,\ 0)$	$(0,\ 6)$	Positive
E	$2y + x = 4$	$(4,\ 0)$	$(0,\ 2)$	Negative
F	$2x + y - 3 = 0$	$(\frac{3}{2},\ 0)$	$(0,\ 3)$	Negative

A pair of lines that are parallel	A and D as they do not intersect
A pair of lines that are perpendicular	A and E or D and E
A pair of lines that intersect at $(-2,\ 2)$	C and D

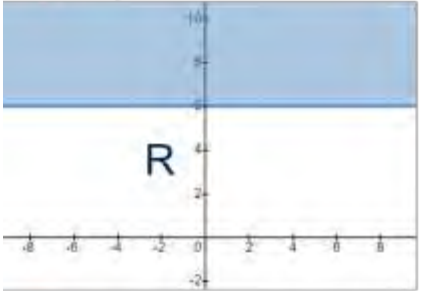
Two geometry problems

The equation of line DE	$y = 3x - 15$	The coordinate of C	$(7,7)$
The possible coordinates of F	$(3,4)$ *or* $(9,2)$	The equation of line AB	$y = 4$
The equation of line EF	$y = \frac{1}{2}x - \frac{5}{2}$ or $y = -2x + 10$	The equation of line BD	$y = -\frac{3}{2}x + 10$
		The area of the parallelogram	15 units^2

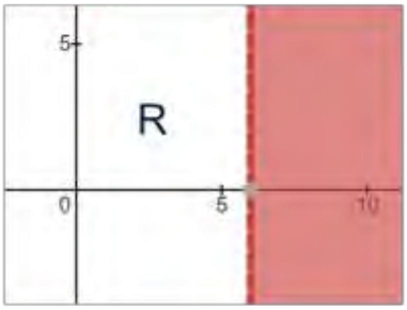
Sketching Solutions

Sketching Linear Inequalities

1. $y \leq 6$

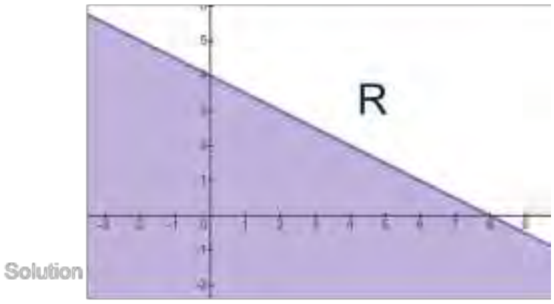


2. $x < 6$



3. $x + 2y \geq 8$

4. $3x + 2y \geq 12$



Geometry from equations

Which are the two pairs of parallel sides? $y - 2 = x$ with $y = x - 1$ and $y + x = 6$ with $y + x - 3 = 0$

What are the coordinates of all 4 vertices? (0.5, 2.5) (2, 4) (3.5, 2.5) (2, 1)

How can you convince yourself this is a square? As well as all the lines that meet being perpendicular, you also need to show they all have the same length. You can do this by using Pythagoras' theorem, or calculating the column vector.

Linear Programming

To maximise the value of $x + y$ within the feasible region, we substitute in the coordinates of each vertex.

$(0,6)\quad x + y = 0 + 6 = 6$

$(2,3)\quad x + y = 2 + 3 = 5$

$(6,6)\quad x + y = 6 + 6 = 12$

$(6,1)\quad x + y = 6 + 1 = 7$

So the **maximum value of $x + y$ is 12** at the point (6,6)

Quadratic Sketching

Sketching Solutions

Quadratic Graphs 1

1\.
A $(2,0)$
B $(-3,0)$
C $(0,-6)$

2\.
A $(3,0)$
B $(-2,0)$
C $(0,-6)$

3\.
A $(5,0)$
B $(-1,0)$
C $(0,-5)$

4\.
A $(4,0)$
B $(-5,0)$
C $(0,-20)$

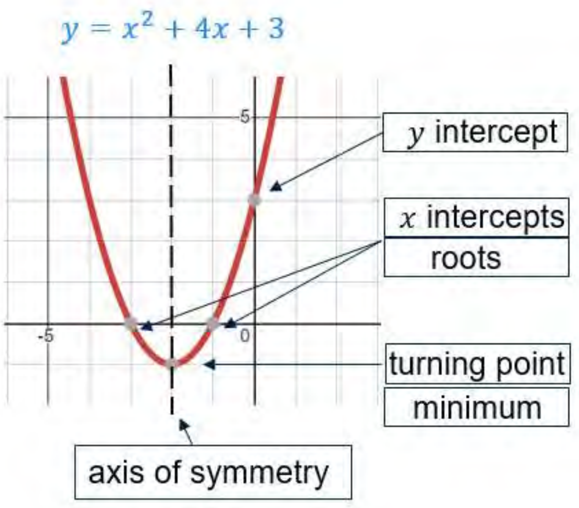
5\.
C $(0,5)$
D $(1,4)$

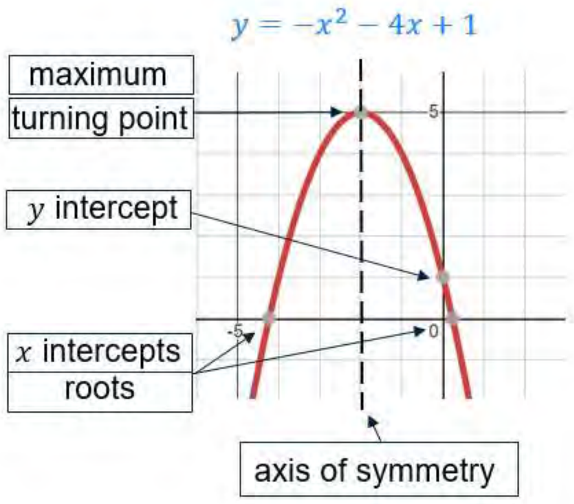
6\.
C $(0,16)$
D $(-3,7)$

7\.
A $(1,0)$
B $(-4,0)$
C $(0,-4)$

8\.
A $(1,0)$
B $(-3,0)$
C $(0,-6)$
D $(-1,-8)$

What is a sketch?





Identification Parade

Graph C is $\boldsymbol{y = x^2 - 5x + 4}$**?**

Move it!

Graph A – Graph B. Translate by the vector $\begin{pmatrix} \mathbf{0} \\ \mathbf{-1} \end{pmatrix}$

Graph A – Graph C. Translate by the vector $\begin{pmatrix} \mathbf{0} \\ \mathbf{3} \end{pmatrix}$

Graph A – Graph D. Translate by the vector $\begin{pmatrix} \mathbf{4} \\ \mathbf{0} \end{pmatrix}$

Graph A – Graph E. Translate by the vector $\begin{pmatrix} \mathbf{2} \\ \mathbf{0} \end{pmatrix}$

Graph A – Graph F. Translate by the vector $\begin{pmatrix} \mathbf{4} \\ \mathbf{3} \end{pmatrix}$

Sketching Solutions

Complete the square to get sorted!

$y = x^2 + 8x + 20$ $y = (x+4)^2 + 4$ $(-4,4)$	$y = x^2 - 2x + 5$ $y = (x-1)^2 + 4$ $(1,4)$	$y = x^2 - 10x + 29$ $y = (x-5)^2 + 4$ $(5,4)$
$y = x^2 + 6x + 10$ $y = (x+3)^2 + 1$ $(-3,1)$	$y = x^2 - 4x + 7$ $y = (x-2)^2 + 3$ $(2,3)$	$y = x^2 + 2x + 5$ $y = (x+1)^2 + 4$ $(-1,4)$
$y = x^2 - 6x + 16$ $y = (x-3)^2 + 7$ $(3,7)$	$y = x^2 - 6x + 11$ $y = (x-3)^2 + 2$ $(3,2)$	$y = x^2 - 6x + 25$ $y = (x-3)^2 + 16$ $(3,16)$

Quadratic Graphs 2

1. x intercepts are $\left(-\frac{3}{2},\ 0\right)$ and $(-4,0)$

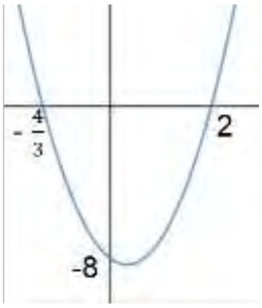
2. x intercepts are $(-3,\ 0)$and $(2,0)$
y intercepts is $(0,-6)$

3. $y = x^2 - x\ - 6$

4. x intercepts are $\left(-\frac{2}{3},0\right)$and $(\frac{1}{2},0)$

5. The y intercept. Coordinate $(0,c)$

6.



7. C is (0,16) D is $(-3,7)$

8.

Has a minimum point at (2, 4)

Will not cross the x axis twice

Sketching Solutions

How High?

The ball hits the ground when t=2 (after 2 seconds)

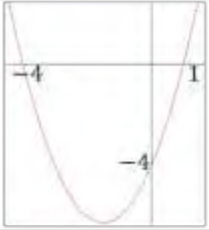
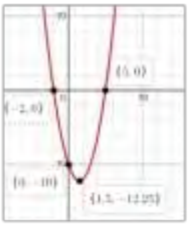
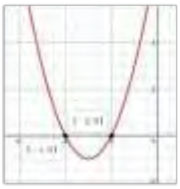
The ball is more than 5m above the ground for 1.414 seconds (to 2dp)

The maximum height reached by the ball is 10 metres

Quadratic Inequalities

1. $-3 < x < 2$
2. $x < -4$ or $x > 2$
3. $x \leq -4$ or $x \geq -3$
4. $-8 \leq x \leq 4$

Fill the table

Sketch	Equation	x intercept	y intercept	Minimum point
	$y=(x+4)(x-1)$ $y=x^2+3x-4$	$(-4,0)(1,0)$	$(0,-4)$	$\left(x+\frac{3}{2}\right)^2-\frac{9}{4}-4$ $\left(x+\frac{3}{2}\right)^2-\frac{25}{4}$ Min point $(-\frac{3}{2},-\frac{25}{4})$
	$y=(x-5)(x+2)$ $y=x^2-3x-10$	$(5,0)\ (-2,0)$	$(0,-10)$	$\left(x-\frac{3}{2}\right)^2-\frac{9}{4}-10$ $\left(x-\frac{3}{2}\right)^2-\frac{49}{4}$ Min point $(\frac{3}{2},-\frac{49}{4})$
	$y=(x+5)^2+6$ $y=x^2+10x+31$	Sits above x axis so no intercepts	$(0,31)$	(-5, 6)
	$y=x^2+6x+8$	$(-4,0)(-2,0)$	$(0,8)$	$(x+3)^2-9+8$ $(x+3)^2-1$ Min point $(-3,-1)$

Sketching Solutions

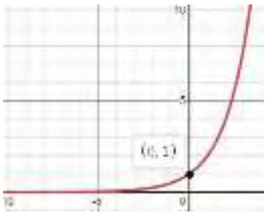
Sketching Other Graphs

Sketching Other Graphs 1

1. A reciprocal Graph

2. Max value = 1 Min value = -1

3. As x gets very large y gets very large. As x gets very small, y tends to zero but stays positive.



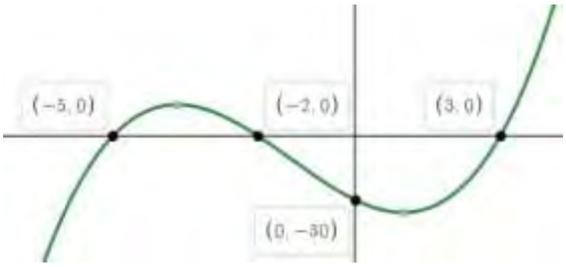
4. There will be two solutions

5. A cubic

6. y intercept at $(0,-30)$

7. x intercepts at $(-2,0)$ $(3,0)$ $(-5,0)$

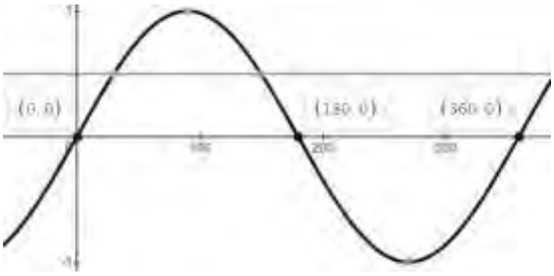
8.



Sketching Other Graphs 2

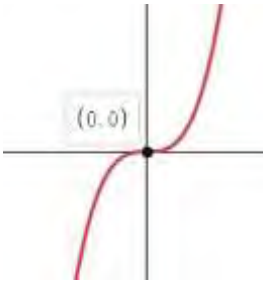
1. Circles are of the form $x^2 + y^2 = r^2$

2.

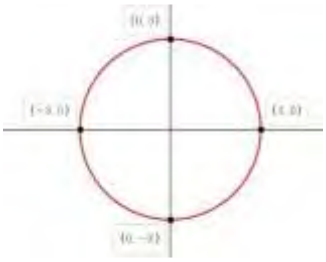


3. Two solutions $30°$ and $150°$, the points of intersection above

4.



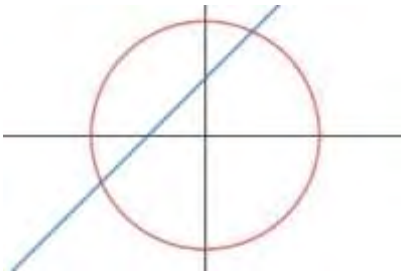
5.



6. y intercept is $(0,-1)$

7. x intercept is $(-1,0)$ repeated and $(1, 0)$

8. Two solutions, where line crosses circle



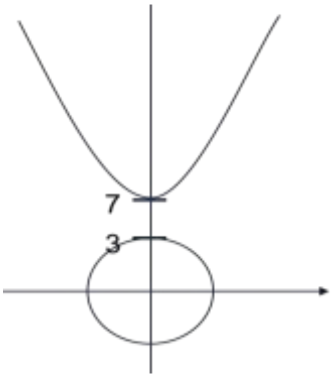
Sketching Solutions

Which is which?

A $y = 3^x$ **B** $y = x^2$ **C** $y^2 + x^2 = 16$ **D** $y = \frac{3}{x}$

E $y = x^3$ **F** $y = \sin{(x)}$

Shortest distance



From the sketch we can see that the graphs are separated at the two marked points and this will be the shortest distance, which is 4 units

How fast?

a) 350m/min

b) 800m/min

A square in a circle

The ratio of C_1 to C_2 is $\sqrt{\mathbf{2}} : \mathbf{1}$

A Triggy Problem!

Fortunately, this is an already factorised quadratic. So, $x = 270°$ or $x = 60°\ and\ 300°$

A cubic match up

$\boldsymbol{y = x(x - 1)\ (x + 1)}$